

# Package ‘PreciseSums’

September 18, 2024

**Title** Accurate Floating Point Sums and Products

**Version** 0.7

**Description** Most of the time floating point arithmetic does approximately the right thing. When adding sums or having products of numbers that greatly differ in magnitude, the floating point arithmetic may be incorrect. This package implements the Kahan (1965) sum <[doi:10.1145/363707.363723](https://doi.org/10.1145/363707.363723)>, Neumaier (1974) sum <[doi:10.1002/zamm.19740540106](https://doi.org/10.1002/zamm.19740540106)>, pairwise-sum (adapted from 'NumPy', See Castaldo (2008) <[doi:10.1137/070679946](https://doi.org/10.1137/070679946)> for a discussion of accuracy), and arbitrary precision sum (adapted from the fsun in 'Python' ; Shewchuk (1997) <<https://people.eecs.berkeley.edu/~jrs/papers/robustr.pdf>>). In addition, products are changed to long double precision for accuracy, or changed into a log-sum for accuracy.

**Depends** R (>= 3.2)

**License** GPL (>= 2)

**Encoding** UTF-8

**RoxygenNote** 7.3.2

**Suggests** testthat

**BugReports** <https://github.com/nlmixr2/PreciseSums/issues/>

**URL** <https://github.com/nlmixr2/PreciseSums>

**NeedsCompilation** yes

**Author** Matthew Fidler [aut, cre, cph],  
Raymond Hettinger [cph, aut],  
Jonathan Shewchuk [cph, aut],  
Julian Taylor [cph, aut],  
Nathaniel Smith [cph, aut],  
NumPy Team [cph],  
Python Team [cph]

**Maintainer** Matthew Fidler <[matthew.fidler@gmail.com](mailto:matthew.fidler@gmail.com)>

**Repository** CRAN

**Date/Publication** 2024-09-17 22:20:12 UTC

## Contents

fsum . . . . .	2
kahanSum . . . . .	3
neumaierSum . . . . .	3
pairwiseSum . . . . .	4
psProd . . . . .	5
psSetProd . . . . .	5
psSetSum . . . . .	6
psSum . . . . .	6
<b>Index</b>	<b>7</b>

---

fsum	<i>Return an accurate floating point sum of values</i>
------	--

---

### Description

This method avoids loss of precision by tracking multiple intermediate partial sums. Based on python's math.fsum

### Usage

```
fsum(numbers)
```

### Arguments

numbers            A vector of numbers to sum.

### Value

Sum of numbers without loss of precision

The algorithm's accuracy depends on IEEE-754 arithmetic guarantees and the typical case where the rounding mode is half-even. On some non-Windows builds, the underlying C library uses extended precision addition and may occasionally double-round an intermediate sum causing it to be off in its least significant bit.

### Author(s)

Matthew Fidler (R implementation), Raymond Hettinger, Jonathan Shewchuk, Python Team

### References

<https://docs.python.org/2/library/math.html> <https://github.com/python/cpython/blob/a0ce375e10b50f7606cb86b072fed7d8cd574fe7/Modules/mathmodule.c>

Shewchuk, JR. (1996) *Adaptive Precision Floating-Point Arithmetic and Fast Robust Geometric Predicates*. <http://www-2.cs.cmu.edu/afs/cs/project/quake/public/papers/robust-arithmetic.ps>

**Examples**

```
sum(c(1,1e100,1,-1e100)) ## Should be 2, gives 0
fsum(c(1,1e100,1,-1e100)) ## Gives 2.
```

---

kahanSum	<i>Using the Kahan method, take a more accurate sum</i>
----------	---

---

**Description**

Using the Kahan method, take a more accurate sum

**Usage**

```
kahanSum(numbers)
```

**Arguments**

numbers            A vector of numbers to sum.

**Value**

Sum of numbers

**References**

[https://en.wikipedia.org/wiki/Kahan\\_summation\\_algorithm](https://en.wikipedia.org/wiki/Kahan_summation_algorithm)

**Examples**

```
sum(c(1,1e100,1,-1e100)) ## Should be 2, gives 0
kahanSum(c(1,1e100,1,-1e100)) ## Not accurate enough for the correct result. (still = 0)
```

---

neumaierSum	<i>Using the Neumaier method, take a more accurate sum</i>
-------------	--

---

**Description**

Using the Neumaier method, take a more accurate sum

**Usage**

```
neumaierSum(numbers)
```

**Arguments**

numbers            A vector of numbers to sum.

**Value**

Sum of numbers, a bit more accurate than kahanSum

**References**

[https://en.wikipedia.org/wiki/Kahan\\_summation\\_algorithm](https://en.wikipedia.org/wiki/Kahan_summation_algorithm)

**Examples**

```
sum(c(1,1e100,1,-1e100)) ## Should be 2, gives 0
neumaierSum(c(1,1e100,1,-1e100)) ## Gives 2
```

---

pairwiseSum

*Return an accurate floating point sum of values*

---

**Description**

This was taken by NumPy and adapted for use here. It is more accurate than a standard sum, but still has numerical issues. Its main benefit is that it is about the same amount of time as a standard time with the added accuracy.

**Usage**

```
pairwiseSum(numbers)
```

**Arguments**

numbers            A vector of numbers to sum.

**Value**

A sum of numbers with a rounding error of  $O(\lg n)$  instead of  $O(n)$ .

**Author(s)**

Matthew Fidler (R implementation), Julian Taylor, Nathaniel J Smith, and NumPy team. #’ @examples sum(c(1,1e100,1,-1e100)) ## Should be 2, gives 0 pairwiseSum(c(1,1e100,1,-1e100)) ## Should be 2, still 0

**References**

<https://github.com/juliantaylor/numpy/blob/b0bc01275cac04483e6df021211c1fa2ba65eaa3/numpy/core/src/umath/loops.c.src>

<https://github.com/numpy/numpy/pull/3685>

---

psProd	<i>Using PreciceSums's default method, take a product</i>
--------	---

---

**Description**

Using PreciceSums's default method, take a product

**Usage**

```
psProd(numbers)
```

**Arguments**

numbers	A vector of numbers to sum.
---------	-----------------------------

**Value**

Product of numbers

---

psSetProd	<i>Choose the type of product to use in PreciceSums. These are used in the PreciceSums prod blocks</i>
-----------	--

---

**Description**

Choose the type of product to use in PreciceSums. These are used in the PreciceSums prod blocks

**Usage**

```
psSetProd(type = c("long double", "double", "logify"))
```

**Arguments**

type	Product to use for prod() in PreciceSums blocks long double converts to long double, performs the multiplication and then converts back. double uses the standard double scale for multiplication.
------	--

**Value**

nothing

**Author(s)**

Matthew L. Fidler

---

psSetSum *Choose the type of sums to use for PreciceSums.*

---

### Description

Choose the types of sums to use in PreciceSums. These are used in the PreciceSums sum blocks and the psSum function

### Usage

```
psSetSum(type = c("pairwise", "fsum", "kahan", "neumaier", "klein", "c"))
```

### Arguments

type	Sum type to use for psSum and sum() in PreciceSums code blocks. pairwise uses the pairwise sum (fast, default) fsum uses Python's fsum function (most accurate) kahan uses kahan correction neumaier uses Neumaier correction klein uses Klien correction c uses no correction, but default/native summing
------	--

### Value

nothing

### Author(s)

Matthew L. Fidler

---

psSum *Using PreciceSums's default method, take a sum*

---

### Description

Using PreciceSums's default method, take a sum

### Usage

```
psSum(numbers)
```

### Arguments

numbers	A vector of numbers to sum.
---------	-----------------------------

### Value

Sum of numbers

# Index

fsum, [2](#)

kahanSum, [3](#)

neumaierSum, [3](#)

pairwiseSum, [4](#)

psProd, [5](#)

psSetProd, [5](#)

psSetSum, [6](#)

psSum, [6](#)