Package ‘fitODBOD’

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Type Package

Title Modeling Over Dispersed Binomial Outcome Data Using BMD and ABD

Version 1.5.4

Description Contains Probability Mass Functions, Cumulative Mass Functions, Negative Log Likelihood value, parameter estimation and modeling data using Binomial Mixture Distributions (BMD) (Manoj et al (2013) <doi:10.5539/ijsp.v2n2p24>) and Alternate Binomial Distributions (ABD) (Paul (1985) <doi:10.1080/03610928508828990>), also Journal article to use the package(<doi:10.21105/joss.01505>).

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URL https://github.com/Amalan-ConStat/fitODBOD,https://amalan-constat.github.io/fitODBOD/index.html,https://amalan-con-stat.shinyapps.io/fitODBODRshiny/

BugReports https://github.com/Amalan-ConStat/fitODBOD/issues

Depends R (>= 4.0.0)

Imports bbmle, hypergeo, MASS, mvtnorm, Rdpack, stats

RdMacros Rdpack

Suggests flextable, ggplot2, ggthemes, grid, gridExtra, knitr, reshape2, rmarkdown, spelling, testthat, tibble, viridis

Encoding UTF-8

Language en-US

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NeedsCompilation no

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Repository CRAN

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Contents

`Alcohol_data` *Alcohol data*

Description

Lemmens , Knibbe and Tan(1988) described a study of self reported alcohol frequencies. The no of alcohol consumption data in two reference weeks is separately self reported by a randomly selected sample of 399 respondents in the Netherlands in 1983. Number of days a given individual consumes alcohol out of 7 days a week can be treated as a binomial variable. The collection of all such variables from all respondents would be defined as "Binomial Outcome Data".

Usage

```
Alcohol_data
```

Format

A data frame with 3 columns and 8 rows.

`Days` No of Days Drunk

`week1` Observed frequencies for week1

`week2` Observed frequencies for week2

Source

Extracted from

Manoj, C., Wijekoon, P. & Yapa, R.D., 2013. The McDonald Generalized Beta-Binomial Distribution: A New Binomial Mixture Distribution and Simulation Based Comparison with Its Nested Distributions in Handling Overdispersion. International Journal of Statistics and Probability, 2(2), pp.24-41.

Available at: doi:10.5539/ijsp.v2n2p24

Examples

```
Alcohol_data$Days          # extracting the binomial random variables
sum(Alcohol_data$week2)       # summing all the frequencies in week2
```

BODextract *Binomial Data Extraction from Raw data*

Description

The below function has the ability to extract from the raw data to Binomial Outcome Data. This function simplifies the data into more presentable way to the user.

Usage

```
BODextract(data)
```

Arguments

data	vector of observations

Details

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further

Value

The output of `BODextract` gives a list format consisting

`RV` binomial random variables in vector form

`Freq` corresponding frequencies in vector form

Examples

```
datapoints <- sample(0:10,340,replace=TRUE) #creating a sample set of observations
BODextract(datapoints)              #extracting binomial outcome data from observations
Random.variable <- BODextract(datapoints)$RV #extracting the binomial random variables
```

Chromosome_data *Chromosome Data*

Description

Data in this example refer to 337 observations on the secondary association of chromosomes in Brassika; n , which is now the number of chromosomes, equals 3 and X is the number of pairs of bivalents showing association.

Usage

```
Chromosome_data
```

Format

A data frame with 2 columns and 4 rows

`No.of.Asso` No of Associations

`fre` Observed frequencies

Source

Extracted from

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi:10.1080/03610928508828990

Examples

```
Chromosome_data$No.of.Asso          #extracting the binomial random variables
sum(Chromosome_data$fre)            #summing all the frequencies
```

`Course_data` *Course Data*

Description

The data refer to the numbers of courses taken by a class of 65 students from the first year of the Department of Statistics of Athens University of Economics. The students enrolled in this class attended 8 courses during the first year of their study. The total numbers of successful examinations (including resits) were recorded.

Usage

```
Course_data
```

Format

A data frame with 2 columns and 9 rows

`sub.pass` subjects passed

`fre` Observed frequencies

Source

Extracted from

Karlis, D. & Xekalaki, E., 2008. The Polygonal Distribution. In Advances in Mathematical and Statistical Modeling. Boston: Birkhuser Boston, pp. 21-33.

Available at: doi:10.1007/9780817646264_2.

Examples

```
Course_data$sub.pass            # extracting the binomial random variables
sum(Course_data$fre)            # summing all the frequencies
```

`dAddBin` *Additive Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Additive Binomial Distribution.

Usage

```
dAddBin(x,n,p,alpha)
```

Arguments

`x`	vector of binomial random variables.
`n`	single value for no of binomial trials.
`p`	single value for probability of success
`alpha`	single value for alpha parameter.

Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{AddBin}(x) = {n \choose x} p^x (1-p)^{n-x} (\frac{alpha}{2}(\frac{x(x-1)}{p} + \frac{(n-x)(n-x-1)}{(1-p)} - \frac{alpha(n-1)n}{2}) + 1)$$

The alpha is in between

$$\frac{-2}{n(n-1)} min(\frac{p}{1-p}, \frac{1-p}{p}) \leq alpha \leq (\frac{n+(2p-1)^2}{4p(1-p)})^{-1}$$

$$x = 0, 1, 2, 3, ...n$$
$$n = 1, 2, 3, ...$$
$$0 < p < 1$$
$$-1 < alpha < 1$$

The mean and the variance are denoted as

$$E_{Addbin}[x] = np$$
$$Var_{Addbin}[x] = np(1-p)(1+(n-1)alpha)$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dAddBin` gives a list format consisting

`pdf` probability function values in vector form.

`mean` mean of Additive Binomial Distribution.

`var` variance of Additive Binomial Distribution.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). "The use of a correlated binomial model for the analysis of certain toxicological experiments." *Biometrics*, 69–76. Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506. Morel JG, Neerchal NK (2012). *Overdispersion models in SAS*. SAS Publishing.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Additive binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dAddBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dAddBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}

dAddBin(0:10,10,0.58,0.022)$pdf     #extracting the probability values
dAddBin(0:10,10,0.58,0.022)$mean    #extracting the mean
dAddBin(0:10,10,0.58,0.022)$var     #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Additive binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pAddBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pAddBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}

pAddBin(0:10,10,0.58,0.022)       #acquiring the cumulative probability values
```

dBETA *Beta Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Beta Distribution bounded between [0,1]

Usage

```
dBETA(p,a,b)
```

Arguments

p	vector of probabilities.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.

Details

The probability density function and cumulative density function of a unit bounded Beta distribution with random variable P are given by

$$g_P(p) = \frac{p^{a-1}(1-p)^{b-1}}{B(a,b)}$$

; $0 \leq p \leq 1$

$$G_P(p) = \frac{B_p(a,b)}{B(a,b)}$$

; $0 \leq p \leq 1$

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{a}{a+b}$$

$$var[P] = \frac{ab}{(a+b)^2(a+b+1)}$$

The moments about zero is denoted as

$$E[P^r] = \prod_{i=0}^{r-1}(\frac{a+i}{a+b+i})$$

$r = 1, 2, 3, ...$

Defined as $B_p(a,b) = \int_0^p t^{a-1}(1-t)^{b-1}\, dt$ is incomplete beta integrals and $B(a,b)$ is the beta function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dBETA` gives a list format consisting

`pdf` probability density values in vector form.

`mean` mean of the Beta distribution.

`var` variance of the Beta distribution.

References

Johnson NL, Kotz S, Balakrishnan N (1995). *Continuous univariate distributions, volume 2*, volume 289. John wiley and sons. Trenkler G (1996). "Continuous univariate distributions." *Computational Statistics and Data Analysis*, **21**(1), 119–119.

See Also

`Beta`

or

https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Beta.html

Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dBETA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dBETA(seq(0,1,by=0.01),2,3)$pdf   #extracting the pdf values
dBETA(seq(0,1,by=0.01),2,3)$mean  #extracting the mean
dBETA(seq(0,1,by=0.01),2,3)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pBETA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pBETA(seq(0,1,by=0.01),2,3)   #acquiring the cumulative probability values
mazBETA(1.4,3,2)              #acquiring the moment about zero values
mazBETA(2,3,2)-mazBETA(1,3,2)^2 #acquiring the variance for a=3,b=2

#only the integer value of moments is taken here because moments cannot be decimal
```

```
mazBETA(1.9,5.5,6)
```

`dBetaBin` *Beta-Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Beta-Binomial Distribution.

Usage

```
dBetaBin(x,n,a,b)
```

Arguments

`x`	vector of binomial random variables.
`n`	single value for no of binomial trials.
`a`	single value for shape parameter alpha representing as a.
`b`	single value for shape parameter beta representing as b.

Details

Mixing Beta distribution with Binomial distribution will create the Beta-Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{BetaBin}(x) = \binom{n}{x} \frac{B(a+x,n+b-x)}{B(a,b)}$$

$$a,b > 0$$

$$x = 0,1,2,3,...n$$

$$n = 1,2,3,...$$

The mean, variance and over dispersion are denoted as

$$E_{BetaBin}[x] = \frac{na}{a+b}$$

$$Var_{BetaBin}[x] = \frac{(nab)}{(a+b)^2} \frac{(a+b+n)}{(a+b+1)}$$

$$overdispersion = \frac{1}{a+b+1}$$

Defined as `B(a,b)` is the beta function.

Value

The output of dBetaBin gives a list format consisting

pdf probability function values in vector form.

mean mean of the Beta-Binomial Distribution.

var variance of the Beta-Binomial Distribution.

over.dis.para over dispersion value of the Beta-Binomial Distribution.

References

Young-Xu Y, Chan KA (2008). “Pooling overdispersed binomial data to estimate event rate.” *BMC medical research methodology*, **8**, 1–12. Trenkler G (1996). “Continuous univariate distributions.” *Computational Statistics and Data Analysis*, **21**(1), 119–119. HUGHES G, MADDEN L (1993). “Using the beta-binomial distribution to describe aggegated patterns of disease incidence.” *Phytopathology*, **83**(7), 759–763.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.2)
plot(0,0,main="Beta-binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dBetaBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dBetaBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

dBetaBin(0:10,10,4,.2)$pdf    #extracting the pdf values
dBetaBin(0:10,10,4,.2)$mean   #extracting the mean
dBetaBin(0:10,10,4,.2)$var    #extracting the variance
dBetaBin(0:10,10,4,.2)$over.dis.para  #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pBetaBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pBetaBin(0:10,10,a[i],a[i]),col = col[i])
}

pBetaBin(0:10,10,4,.2)   #acquiring the cumulative probability values
```

`dBetaCorrBin` *Beta-Correlated Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Beta-Correlated Binomial Distribution.

Usage

```
dBetaCorrBin(x,n,cov,a,b)
```

Arguments

`x`	vector of binomial random variables.
`n`	single value for no of binomial trials.
`cov`	single value for covariance.
`a`	single value for alpha parameter.
`b`	single value for beta parameter.

Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{BetaCorrBin}(x) = \binom{n}{x} \frac{B(a+x,b+n-x)}{B(a+b)} \left[1 + \frac{cov}{2}\left(\frac{\left(x(x-1)\prod_{k=1}^{4}(a+b+n-k)\right)}{\left(\prod_{k=1}^{2}(x+a-k)\prod_{k=1}^{2}(n-x+b-k)\right)} - \frac{\left(2x(n-1)\prod_{k=1}^{3}(a+b+n-k)\right)}{\left((x+a-1)\prod_{k=1}^{2}(n-x+b-k)\right)} + \frac{\left(n(n-1)\prod_{k=1}^{2}(a+b+n-k)\right)}{\left(\prod_{k=1}^{2}(n-x+b-k)\right)}\right)\right]$$

$$x = 0, 1, 2, 3, ...n$$

$$n = 1, 2, 3, ...$$

$$0 < a, b$$

$$-\infty < cov < +\infty$$

$$0 < p < 1$$

$$p = \frac{a}{a+b}$$

$$\Theta = \frac{1}{a+b}$$

The Correlation is in between

$$\frac{-2}{n(n-1)}min(\frac{p}{1-p},\frac{1-p}{p}) \leq correlation \leq \frac{2p(1-p)}{(n-1)p(1-p)+0.25-fo}$$

where $fo = min[(x-(n-1)p-0.5)^2]$

The mean and the variance are denoted as

$$E_{BetaCorrBin}[x] = np$$

$$Var_{BetaCorrBin}[x] = np(1-p)(n\Theta+1)(1+\Theta)^{-1} + n(n-1)cov$$

$$Corr_{BetaCorrBin}[x] = \frac{cov}{p(1-p)}$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dBetaCorrBin` gives a list format consisting

`pdf` probability function values in vector form.

`mean` mean of Beta-Correlated Binomial Distribution.

`var` variance of Beta-Correlated Binomial Distribution.

`corr` correlation of Beta-Correlated Binomial Distribution.

`mincorr` minimum correlation value possible.

`maxcorr` maximum correlation value possible.

References

Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(9.0,10,11,12,13)
b <- c(8.0,8.1,8.2,8.3,8.4)
plot(0,0,main="Beta-Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dBetaCorrBin(0:10,10,0.001,a[i],b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dBetaCorrBin(0:10,10,0.001,a[i],b[i])$pdf,col = col[i],pch=16)
}

dBetaCorrBin(0:10,10,0.001,10,13)$pdf      #extracting the pdf values
dBetaCorrBin(0:10,10,0.001,10,13)$mean     #extracting the mean
dBetaCorrBin(0:10,10,0.001,10,13)$var      #extracting the variance
```

```
dBetaCorrBin(0:10,10,0.001,10,13)$corr     #extracting the correlation
dBetaCorrBin(0:10,10,0.001,10,13)$mincorr  #extracting the minimum correlation value
dBetaCorrBin(0:10,10,0.001,10,13)$maxcorr  #extracting the maximum correlation value

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(9.0,10,11,12,13)
b <- c(8.0,8.1,8.2,8.3,8.4)
plot(0,0,main="Beta-Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pBetaCorrBin(0:10,10,0.001,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pBetaCorrBin(0:10,10,0.001,a[i],b[i]),col = col[i],pch=16)
}

pBetaCorrBin(0:10,10,0.001,10,13)      #acquiring the cumulative probability values
```

`dCOMPBin` *COM Poisson Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the COM Poisson Binomial Distribution.

Usage

```
dCOMPBin(x,n,p,v)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
p	single value for probability of success.
v	single value for v.

Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{COMPBin}(x) = \frac{\binom{n}{x}^{v} p^x (1-p)^{n-x}}{\sum_{j=0}^{n} \binom{n}{j}^{v} p^j (1-p)^{(n-j)}}$$

$$x = 0, 1, 2, 3, ...n$$

$$n = 1, 2, 3, ...$$

$$0 < p < 1$$

$$-\infty < v < +\infty$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dCOMPBin` gives a list format consisting

`pdf` probability function values in vector form.

`mean` mean of COM Poisson Binomial Distribution.

`var` variance of COM Poisson Binomial Distribution.

References

Borges P, Rodrigues J, Balakrishnan N, Bazan J (2014). "A COM–Poisson type generalization of the binomial distribution and its properties and applications." *Statistics and Probability Letters*, **87**, 158–166.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="COM Poisson Binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dCOMPBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dCOMPBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}

dCOMPBin(0:10,10,0.58,0.022)$pdf      #extracting the pdf values
dCOMPBin(0:10,10,0.58,0.022)$mean     #extracting the mean
dCOMPBin(0:10,10,0.58,0.022)$var      #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="COM Poisson Binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pCOMPBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pCOMPBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}
```

```
pCOMPBin(0:10,10,0.58,0.022)      #acquiring the cumulative probability values
```

`dCorrBin` *Correlated Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Correlated Binomial Distribution.

Usage

```
dCorrBin(x,n,p,cov)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
p	single value for probability of success.
cov	single value for covariance.

Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{CorrBin}(x) = \binom{n}{x}(p^x)(1-p)^{n-x}(1+(\frac{cov}{2p^2(1-p)^2})((x-np)^2+x(2p-1)-np^2))$$

$$x = 0, 1, 2, 3, ...n$$

$$n = 1, 2, 3, ...$$

$$0 < p < 1$$

$$-\infty < cov < +\infty$$

The Correlation is in between

$$\frac{-2}{n(n-1)}min(\frac{p}{1-p}, \frac{1-p}{p}) \leq correlation \leq \frac{2p(1-p)}{(n-1)p(1-p)+0.25-fo}$$

where $fo = min[(x-(n-1)p-0.5)^2]$

The mean and the variance are denoted as

$$E_{CorrBin}[x] = np$$

$$Var_{CorrBin}[x] = n(p(1-p)+(n-1)cov)$$

$$Corr_{CorrBin}[x] = \frac{cov}{p(1-p)}$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dCorrBin` gives a list format consisting

`pdf` probability function values in vector form.

`mean` mean of Correlated Binomial Distribution.

`var` variance of Correlated Binomial Distribution.

`corr` correlation of Correlated Binomial Distribution.

`mincorr` minimum correlation value possible.

`maxcorr` maximum correlation value possible.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). "The use of a correlated binomial model for the analysis of certain toxicological experiments." *Biometrics*, 69–76. Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506. Morel JG, Neerchal NK (2012). *Overdispersion models in SAS*. SAS Publishing.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dCorrBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dCorrBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}

dCorrBin(0:10,10,0.58,0.022)$pdf      #extracting the pdf values
dCorrBin(0:10,10,0.58,0.022)$mean     #extracting the mean
dCorrBin(0:10,10,0.58,0.022)$var      #extracting the variance
dCorrBin(0:10,10,0.58,0.022)$corr     #extracting the correlation
dCorrBin(0:10,10,0.58,0.022)$mincorr  #extracting the minimum correlation value
dCorrBin(0:10,10,0.58,0.022)$maxcorr  #extracting the maximum correlation value

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pCorrBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pCorrBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}
```

```
pCorrBin(0:10,10,0.58,0.022)      #acquiring the cumulative probability values
```

---

dGAMMA *Gamma Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for Gamma Distribution bounded between [0,1].

Usage

```
dGAMMA(p,c,l)
```

Arguments

p	vector of probabilities.
c	single value for shape parameter c.
l	single value for shape parameter l.

Details

The probability density function and cumulative density function of a unit bounded Gamma distribution with random variable P are given by

$$g_P(p) = \frac{c^l p^{c-1}}{\gamma(l)}[ln(1/p)]^{l-1}$$

; $0 \leq p \leq 1$

$$G_P(p) = \frac{Ig(l, cln(1/p))}{\gamma(l)}$$

; $0 \leq p \leq 1$

$$l, c > 0$$

The mean the variance are denoted by

$$E[P] = (\frac{c}{c+1})^l$$

$$var[P] = (\frac{c}{c+2})^l - (\frac{c}{c+1})^{2l}$$

The moments about zero is denoted as

$$E[P^r] = (\frac{c}{c+r})^l$$

$r = 1, 2, 3, ...$

Defined as $\gamma(l)$ is the gamma function Defined as $Ig(l, cln(1/p)) = \int_0^{cln(1/p)} t^{l-1}e^{-t}dt$ is the Lower incomplete gamma function

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of dGAMMA gives a list format consisting

pdf probability density values in vector form.

mean mean of the Gamma distribution.

var variance of Gamma distribution.

References

Olshen AC (1938). "Transformations of the pearson type III distribution." *The Annals of Mathematical Statistics*, **9**(3), 176–200.

See Also

GammaDist

Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dGAMMA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dGAMMA(seq(0,1,by=0.01),5,6)$pdf   #extracting the pdf values
dGAMMA(seq(0,1,by=0.01),5,6)$mean  #extracting the mean
dGAMMA(seq(0,1,by=0.01),5,6)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pGAMMA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pGAMMA(seq(0,1,by=0.01),5,6)   #acquiring the cumulative probability values
mazGAMMA(1.4,5,6)              #acquiring the moment about zero values
mazGAMMA(2,5,6)-mazGAMMA(1,5,6)^2 #acquiring the variance for a=5,b=6

#only the integer value of moments is taken here because moments cannot be decimal
mazGAMMA(1.9,5.5,6)
```

`dGammaBin` *Gamma Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Gamma Binomial Distribution.

Usage

```
dGammaBin(x,n,c,l)
```

Arguments

`x`	vector of binomial random variables.
`n`	single value for no of binomial trials.
`c`	single value for shape parameter c.
`l`	single value for shape parameter l.

Details

Mixing Gamma distribution with Binomial distribution will create the the Gamma Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{GammaBin}[x] = \binom{n}{x} \sum_{j=0}^{n-x} \binom{n-x}{j} (-1)^j (\frac{c}{c+x+j})^l$$

$$c, l > 0$$

$$x = 0, 1, 2, ..., n$$

$$n = 1, 2, 3, ...$$

The mean, variance and over dispersion are denoted as

$$E_{GammaBin}[x] = (\frac{c}{c+1})^l$$

$$Var_{GammaBin}[x] = n^2[(\frac{c}{c+2})^l - (\frac{c}{c+1})^{2l}] + n(\frac{c}{c+1})^l 1-)(\frac{c+1}{c+2})^l$$

$$overdispersion = \frac{(\frac{c}{c+2})^l - (\frac{c}{c+1})^{2l}}{(\frac{c}{c+1})^l[1 - (\frac{c}{c+1})^l]}$$

Value

The output of `dGammaBin` gives a list format consisting

`pdf` probability function values in vector form.

`mean` mean of the Gamma Binomial Distribution.

`var` variance of the Gamma Binomial Distribution.

`over.dis.para` over dispersion value of the Gamma Binomial Distribution.

References

Grassia A (1977). "On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions." *Australian Journal of Statistics*, **19**(2), 108–114.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.2)
plot(0,0,main="Gamma Binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dGammaBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dGammaBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

dGammaBin(0:10,10,4,.2)$pdf    #extracting the pdf values
dGammaBin(0:10,10,4,.2)$mean   #extracting the mean
dGammaBin(0:10,10,4,.2)$var    #extracting the variance
dGammaBin(0:10,10,4,.2)$over.dis.para  #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pGammaBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pGammaBin(0:10,10,a[i],a[i]),col = col[i])
}

pGammaBin(0:10,10,4,.2)   #acquiring the cumulative probability values
```

`dGBeta1` *Generalized Beta Type-1 Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Generalized Beta Type-1 Distribution bounded between [0,1].

Usage

```
dGBeta1(p,a,b,c)
```

Arguments

`p`	vector of probabilities.
`a`	single value for shape parameter alpha representing as a.
`b`	single value for shape parameter beta representing as b.
`c`	single value for shape parameter gamma representing as c.

Details

The probability density function and cumulative density function of a unit bounded Generalized Beta Type-1 Distribution with random variable P are given by

$$g_P(p) = \frac{c}{B(a,b)} p^{ac-1} (1-p^c)^{b-1}$$

$; 0 \leq p \leq 1$

$$G_P(p) = \frac{p^{ac}}{aB(a,b)} 2F1(a, 1-b; p^c; a+1)$$

$0 \leq p \leq 1$

$$a, b, c > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$var[P] = \frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - (\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})})^2$$

The moments about zero is denoted as

$$E[P^r] = \frac{B(a+b, \frac{r}{c})}{B(a, \frac{r}{c})}$$

$r = 1, 2, 3, ...$

Defined as $B(a,b)$ is Beta function. Defined as $2F1(a,b;c;d)$ is Gaussian Hypergeometric function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dGBeta1` gives a list format consisting

`pdf` probability density values in vector form.

`mean` mean of the Generalized Beta Type-1 Distribution.

`var` variance of the Generalized Beta Type-1 Distribution.

References

Manoj C, Wijekoon P, Yapa RD (2013). "The McDonald generalized beta-binomial distribution: A new binomial mixture distribution and simulation based comparison with its nested distributions in handling overdispersion." *International journal of statistics and probability*, **2**(2), 24. Janiffer NM, Islam A, Luke O, others (2014). "Estimating Equations for Estimation of Mcdonald Generalized Beta—Binomial Parameters." *Open Journal of Statistics*, **4**(09), 702. Roozegar R, Tahmasebi S, Jafari AA (2017). "The McDonald Gompertz distribution: properties and applications." *Communications in Statistics-Simulation and Computation*, **46**(5), 3341–3355.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
lines(seq(0,1,by=0.001),dGBeta1(seq(0,1,by=0.001),a[i],1,2*a[i])$pdf,col = col[i])
}

dGBeta1(seq(0,1,by=0.01),2,3,1)$pdf    #extracting the pdf values
dGBeta1(seq(0,1,by=0.01),2,3,1)$mean   #extracting the mean
dGBeta1(seq(0,1,by=0.01),2,3,1)$var    #extracting the variance

pGBeta1(0.04,2,3,4)        #acquiring the cdf values for a=2,b=3,c=4
mazGBeta1(1.4,3,2,2)              #acquiring the moment about zero values
mazGBeta1(2,3,2,2)-mazGBeta1(1,3,2,2)^2        #acquiring the variance for a=3,b=2,c=2

#only the integer value of moments is taken here because moments cannot be decimal
mazGBeta1(3.2,3,2,2)
```

dGHGBB *Gaussian Hypergeometric Generalized Beta Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Gaussian Hypergeometric Generalized Beta Binomial distribution.

Usage

```
dGHGBB(x,n,a,b,c)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
a	single value for shape parameter alpha value representing a.
b	single value for shape parameter beta value representing b.
c	single value for shape parameter lambda value representing c.

Details

Mixing Gaussian Hypergeometric Generalized Beta distribution with Binomial distribution will create the Gaussian Hypergeometric Generalized Beta Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{GHGBB}(x) = \frac{1}{2F1(-n,a;-b-n+1;c)} \binom{n}{x} \frac{B(x+a,n-x+b)}{B(a,b+n)} (c^x)$$

$$a,b,c > 0$$

$$x = 0,1,2,...n$$

$$n = 1,2,3,...$$

The mean, variance and over dispersion are denoted as

$$E_{GHGBB}[x] = nE_{GHGBeta}$$

$$Var_{GHGBB}[x] = nE_{GHGBeta}(1-E_{GHGBeta}) + n(n-1)Var_{GHGBeta}$$

$$overdispersion = \frac{var_{GHGBeta}}{E_{GHGBeta}(1-E_{GHGBeta})}$$

Defined as $B(a,b)$ is the beta function. Defined as $2F1(a,b;c;d)$ is the Gaussian Hypergeometric function

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of dGHGBB gives a list format consisting

pdf probability function values in vector form.

mean mean of Gaussian Hypergeometric Generalized Beta Binomial Distribution.

var variance of Gaussian Hypergeometric Generalized Beta Binomial Distribution.

over.dis.para over dispersion value of Gaussian Hypergeometric Generalized Beta Binomial Distribution.

References

Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ, Olmo-Jimenez MJ (2007). "A generalization of the beta–binomial distribution." *Journal of the Royal Statistical Society Series C: Applied Statistics*, **56**(1), 51–61. Pearson JW (2009). *Computation of hypergeometric functions*. Ph.D. thesis, University of Oxford.

See Also

hypergeo_powerseries

Examples

```
#plotting the random variables and probability values
col <- rainbow(6)
a <- c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="GHGBB probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,7),ylim = c(0,0.9))
for (i in 1:6)
{
lines(0:7,dGHGBB(0:7,7,1+a[i],0.3,1+a[i])$pdf,col = col[i],lwd=2.85)
points(0:7,dGHGBB(0:7,7,1+a[i],0.3,1+a[i])$pdf,col = col[i],pch=16)
}

dGHGBB(0:7,7,1.3,0.3,1.3)$pdf      #extracting the pdf values
dGHGBB(0:7,7,1.3,0.3,1.3)$mean     #extracting the mean
dGHGBB(0:7,7,1.3,0.3,1.3)$var      #extracting the variance
dGHGBB(0:7,7,1.3,0.3,1.3)$over.dis.par  #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,7),ylim = c(0,1))
for (i in 1:4)
{
lines(0:7,pGHGBB(0:7,7,1+a[i],0.3,1+a[i]),col = col[i])
points(0:7,pGHGBB(0:7,7,1+a[i],0.3,1+a[i]),col = col[i])
}

pGHGBB(0:7,7,1.3,0.3,1.3)     #acquiring the cumulative probability values
```

dGHGBeta *Gaussian Hypergeometric Generalized Beta Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Gaussian Hypergeometric Generalized Beta distribution bounded between [0,1].

Usage

```
dGHGBeta(p,n,a,b,c)
```

Arguments

p	vector of probabilities.
n	single value for no of binomial trials.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter lambda representing as c.

Details

The probability density function and cumulative density function of a unit bounded Gaussian Hypergeometric Generalized Beta Distribution with random variable P are given by

$$g_P(p) = \frac{1}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1}(1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}}$$

$; 0 \le p \le 1$

$$G_P(p) = \int_0^p \frac{1}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} t^{a-1}(1-t)^{b-1} \frac{c^{b+n}}{(c+(1-c)t)^{a+b+n}} \, dt$$

$; 0 \le p \le 1$

$$a, b, c > 0$$

$$n = 1, 2, 3, ...$$

The mean and the variance are denoted by

$$E[P] = \int_0^1 \frac{p}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1}(1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}} \, dp$$

$$var[P] = \int_0^1 \frac{p^2}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1}(1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}} \, dp - (E[p])^2$$

The moments about zero is denoted as

$$E[P^r] = \int_0^1 \frac{p^r}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1}(1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}}\, dp$$

$r = 1, 2, 3, ...$

Defined as $B(a,b)$ as the beta function. Defined as $2F1(a,b;c;d)$ as the Gaussian Hypergeometric function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dGHGBeta` gives a list format consisting

`pdf` probability density values in vector form.

`mean` mean of the Gaussian Hypergeometric Generalized Beta Distribution.

`var` variance of the Gaussian Hypergeometric Generalized Beta Distribution.

References

Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ, Olmo-Jimenez MJ (2007). "A generalization of the beta–binomial distribution." *Journal of the Royal Statistical Society Series C: Applied Statistics*, **56**(1), 51–61. Pearson JW (2009). *Computation of hypergeometric functions*. Ph.D. thesis, University of Oxford.

See Also

hypergeo_powerseries

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
lines(seq(0,1,by=0.001),dGHGBeta(seq(0,1,by=0.001),7,1+a[i],0.3,1+a[i])$pdf,col = col[i])
}

dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$pdf   #extracting the pdf values
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$mean  #extracting the mean
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(6)
a <- c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
```

```
for (i in 1:6)
{
lines(seq(0.01,1,by=0.001),pGHGBeta(seq(0.01,1,by=0.001),7,1+a[i],0.3,1+a[i]),col=col[i])
}

pGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659) #acquiring the cumulative probability values
mazGHGBeta(1.4,7,1.6312,0.3913,0.6659)           #acquiring the moment about zero values

#acquiring the variance for a=1.6312,b=0.3913,c=0.6659
mazGHGBeta(2,7,1.6312,0.3913,0.6659)-mazGHGBeta(1,7,1.6312,0.3913,0.6659)^2

#only the integer value of moments is taken here because moments cannot be decimal
mazGHGBeta(1.9,15,5,6,1)
```

`dGrassiaIIBin` *Grassia-II-Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Grassia-II-Binomial Distribution.

Usage

```
dGrassiaIIBin(x,n,a,b)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
a	single value for shape parameter a.
b	single value for shape parameter b.

Details

Mixing Gamma distribution with Binomial distribution will create the the Grassia-II-Binomial distribution, only when (1-p)=e^(-lambda) of the Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{GrassiaIIBin}[x] = \binom{n}{x} \sum_{j=0}^{x} \binom{x}{j} (-1)^{x-j} (1 + b(n-j))^{-a}$$

$$a, b > 0$$

$$x = 0, 1, 2, ..., n$$

$$n = 1, 2, 3, ...$$

The mean, variance and over dispersion are denoted as

$$E_{GrassiaIIBin}[x] = (\frac{b}{b+1})^a$$

$$Var_{GrassiaIIBin}[x] = n^2[(\frac{b}{b+2})^a - (\frac{b}{b+1})^{2a}] + n(\frac{b}{b+1})^a 1 - (\frac{b+1}{b+2})^a$$

$$overdispersion = \frac{(\frac{b}{b+2})^l - (\frac{b}{b+1})^{2a}}{(\frac{b}{b+1})^a[1 - (\frac{b}{b+1})^a]}$$

Value

The output of `dGrassiaIIBin` gives a list format consisting

`pdf` probability function values in vector form.

`mean` mean of the Grassia II Binomial Distribution.

`var` variance of the Grassia II Binomial Distribution.

`over.dis.para` over dispersion value of the Grassia II Binomial Distribution.

References

Grassia A (1977). “On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions.” *Australian Journal of Statistics*, **19**(2), 108–114.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.2)
plot(0,0,main="Grassia II binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dGrassiaIIBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dGrassiaIIBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

dGrassiaIIBin(0:10,10,4,.2)$pdf    #extracting the pdf values
dGrassiaIIBin(0:10,10,4,.2)$mean   #extracting the mean
dGrassiaIIBin(0:10,10,4,.2)$var    #extracting the variance
dGrassiaIIBin(0:10,10,4,.2)$over.dis.para  #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <-c (1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
```

```
{
lines(0:10,pGrassiaIIBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pGrassiaIIBin(0:10,10,a[i],a[i]),col = col[i])
}

pGrassiaIIBin(0:10,10,4,.2)   #acquiring the cumulative probability values
```

---

dKUM *Kumaraswamy Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Kumaraswamy Distribution bounded between [0,1].

Usage

```
dKUM(p,a,b)
```

Arguments

p	vector of probabilities.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.

Details

The probability density function and cumulative density function of a unit bounded Kumaraswamy Distribution with random variable P are given by

$$g_P(p) = abp^{a-1}(1-p^a)^{b-1}$$

; $0 \leq p \leq 1$

$$G_P(p) = 1-(1-p^a)^b$$

; $0 \leq p \leq 1$

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = bB(1+\frac{1}{a}, b)$$

$$var[P] = bB(1+\frac{2}{a}, b) - (bB(1+\frac{1}{a}, b))^2$$

The moments about zero is denoted as

$$E[P^r] = bB(1 + \frac{r}{a}, b)$$

$r = 1, 2, 3, ...$

Defined as $B(a, b)$ is the beta function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dKUM` gives a list format consisting

`pdf` probability density values in vector form.

`mean` mean of the Kumaraswamy distribution.

`var` variance of the Kumaraswamy distribution.

References

Kumaraswamy P (1980). "A generalized probability density function for double-bounded random processes." *Journal of hydrology*, **46**(1-2), 79–88. Jones MC (2009). "Kumaraswamy's distribution: A beta-type distribution with some tractability advantages." *Statistical methodology*, **6**(1), 70–81.

Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,6))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dKUM(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dKUM(seq(0,1,by=0.01),2,3)$pdf   #extracting the probability values
dKUM(seq(0,1,by=0.01),2,3)$mean  #extracting the mean
dKUM(seq(0,1,by=0.01),2,3)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pKUM(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pKUM(seq(0,1,by=0.01),2,3)    #acquiring the cumulative probability values
```

```
mazKUM(1.4,3,2)               #acquiring the moment about zero values
mazKUM(2,2,3)-mazKUM(1,2,3)^2  #acquiring the variance for a=2,b=3

#only the integer value of moments is taken here because moments cannot be decimal
mazKUM(1.9,5.5,6)
```

---

dKumBin *Kumaraswamy Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Kumaraswamy Binomial Distribution.

Usage

```
dKumBin(x,n,a,b,it=25000)
```

Arguments

x	vector of binomial random variables
n	single value for no of binomial trial
a	single value for shape parameter alpha representing a
b	single value for shape parameter beta representing b
it	number of iterations to converge as a proper probability function replacing infinity

Details

Mixing Kumaraswamy distribution with Binomial distribution will create the Kumaraswamy Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{KumBin}(x) = ab\binom{n}{x}\sum_{j=0}^{it}(-1)^j\binom{b-1}{j}B(x+a+aj,n-x+1)$$

$$a, b > 0$$

$$x = 0, 1, 2, ...n$$

$$n = 1, 2, 3, ...$$

$$it > 0$$

The mean, variance and over dispersion are denoted as

$$E_{KumBin}[x] = nbB(1 + \frac{1}{a}, b)$$

$$Var_{KumBin}[x] = n^2b(B(1 + \frac{2}{a}, b) - bB(1 + \frac{1}{a}, b)^2) + nb(B(1 + \frac{1}{a}, b) - B(1 + \frac{2}{a}, b))$$

$$overdispersion = \frac{(bB(1 + \frac{2}{a}, b) - (bB(1 + \frac{1}{a}, b))^2)}{(bB(1 + \frac{1}{a}, b) - (bB(1 + \frac{1}{a}, b))^2)}$$

Defined as $B(a, b)$ is the beta function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dKumBin` gives a list format consisting

`pdf` probability function values in vector form.

`mean` mean of the Kumaraswamy Binomial Distribution.

`var` variance of the Kumaraswamy Binomial Distribution.

`over.dis.para` over dispersion value of the Kumaraswamy Distribution.

References

Xiaohu L, Yanyan H, Xueyan Z (2011). “The Kumaraswamy binomial distribution.” *Chinese Journal of Applied Probability and Statistics*, **27**(5), 511–521.

Examples

```
## Not run:
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,.85)
plot(0,0,main="Kumaraswamy binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5) {
lines(0:10,dKumBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dKumBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
  }

## End(Not run)

dKumBin(0:10,10,4,2)$pdf  #extracting the pdf values
dKumBin(0:10,10,4,2)$mean #extracting the mean
dKumBin(0:10,10,4,2)$var  #extracting the variance
dKumBin(0:10,10,4,2)$over.dis.para #extracting the over dispersion value

## Not run:
#plotting the random variables and cumulative probability values
col <- rainbow(5)
```

```
a <- c(1,2,5,10,.85)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5) {
lines(0:10,pKumBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pKumBin(0:10,10,a[i],a[i]),col = col[i])
  }

## End(Not run)

pKumBin(0:10,10,4,2)    #acquiring the cumulative probability values
```

`dLMBin` *Lovinson Multiplicative Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Lovinson Multiplicative Binomial Distribution.

Usage

```
dLMBin(x,n,p,phi)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
p	single value for probability of success.
phi	single value for phi.

Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{LMBin}(x) = {n \choose x} p^x (1-p)^{n-x} \frac{(phi^{x(n-x)}}{f(p,phi,n)}$$

here $f(p, phi, n)$ is

$$f(p,phi,n) = \sum_{k=0}^{n} {n \choose k} p^k (1-p)^{n-k} (phi^{k(n-k)})$$

$$x = 0, 1, 2, 3, ...n$$

$$n = 1, 2, 3, ...$$
$$k = 0, 1, 2, ..., n$$
$$0 < p < 1$$
$$0 < phi$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dLMBin` gives a list format consisting

`pdf` probability function values in vector form.

`mean` mean of Lovinson Multiplicative Binomial Distribution.

`var` variance of Lovinson Multiplicative Binomial Distribution.

References

Elamir EA (2013). "Multiplicative-Binomial Distribution: Some Results on Characterization, Inference and Random Data Generation." *Journal of Statistical Theory and Applications*, **12**(1), 92–105.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Lovinson Multiplicative binomial probability
     function graph",xlab="Binomial random variable",
     ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dLMBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dLMBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],pch=16)
}

dLMBin(0:10,10,.58,10.022)$pdf   #extracting the pdf values
dLMBin(0:10,10,.58,10.022)$mean   #extracting the mean
dLMBin(0:10,10,.58,10.022)$var   #extracting the variance


#plotting random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Lovinson Multiplicative binomial probability
     function graph",xlab="Binomial random variable",
     ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
```

```
lines(0:10,pLMBin(0:10,10,a[i],1+b[i]),col = col[i],lwd=2.85)
points(0:10,pLMBin(0:10,10,a[i],1+b[i]),col = col[i],pch=16)
}

pLMBin(0:10,10,.58,10.022)     #acquiring the cumulative probability values
```

---

dMcGBB *McDonald Generalized Beta Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the McDonald Generalized Beta Binomial Distribution.

Usage

```
dMcGBB(x,n,a,b,c)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter gamma representing as c.

Details

Mixing Generalized Beta Type-1 Distribution with Binomial distribution the probability function value and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{McGBB}(x) = \binom{n}{x} \frac{1}{B(a,b)} (\sum_{j=0}^{n-x} (-1)^j \binom{n-x}{j} B(\frac{x}{c} + a + \frac{j}{c}, b))$$

$$a, b, c > 0$$

The mean, variance and over dispersion are denoted as

$$E_{McGBB}[x] = n \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$Var_{McGBB}[x] = n^2 (\frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - (\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})})^2) + n(\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} - \frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})})$$

$$overdispersion = \frac{\frac{B(a+b,\frac{2}{c})}{B(a,\frac{2}{c})} - (\frac{B(a+b,\frac{1}{c})}{B(a,\frac{1}{c})})^2}{\frac{B(a+b,\frac{1}{c})}{B(a,\frac{1}{c})} - (\frac{B(a+b,\frac{1}{c})}{B(a,\frac{1}{c})})^2}$$

$$x = 0, 1, 2, ...n$$

$$n = 1, 2, 3, ...$$

Value

The output of `dMcGBB` gives a list format consisting

`pdf` probability function values in vector form.

`mean` mean of McDonald Generalized Beta Binomial Distribution.

`var` variance of McDonald Generalized Beta Binomial Distribution.

`over.dis.para` over dispersion value of McDonald Generalized Beta Binomial Distribution.

References

Manoj C, Wijekoon P, Yapa RD (2013). "The McDonald generalized beta-binomial distribution: A new binomial mixture distribution and simulation based comparison with its nested distributions in handling overdispersion." *International journal of statistics and probability*, **2**(2), 24. Janiffer NM, Islam A, Luke O, others (2014). "Estimating Equations for Estimation of Mcdonald Generalized Beta—Binomial Parameters." *Open Journal of Statistics*, **4**(09), 702. Roozegar R, Tahmasebi S, Jafari AA (2017). "The McDonald Gompertz distribution: properties and applications." *Communications in Statistics-Simulation and Computation*, **46**(5), 3341–3355.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.6)
plot(0,0,main="Mcdonald generalized beta-binomial probability function graph",
xlab="Binomial random variable",ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dMcGBB(0:10,10,a[i],2.5,a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dMcGBB(0:10,10,a[i],2.5,a[i])$pdf,col = col[i],pch=16)
}

dMcGBB(0:10,10,4,2,1)$pdf             #extracting the pdf values
dMcGBB(0:10,10,4,2,1)$mean            #extracting the mean
dMcGBB(0:10,10,4,2,1)$var             #extracting the variance
dMcGBB(0:10,10,4,2,1)$over.dis.para   #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
```

```
lines(0:10,pMcGBB(0:10,10,a[i],a[i],2),col = col[i])
points(0:10,pMcGBB(0:10,10,a[i],a[i],2),col = col[i])
}

pMcGBB(0:10,10,4,2,1)       #acquiring the cumulative probability values
```

`dMultiBin` *Multiplicative Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Multiplicative Binomial Distribution.

Usage

```
dMultiBin(x,n,p,theta)
```

Arguments

`x`	vector of binomial random variables.
`n`	single value for no of binomial trials.
`p`	single value for probability of success.
`theta`	single value for theta.

Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{MultiBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} \frac{(theta^{x(n-x)}}{f(p,theta,n)}$$

here $f(p, theta, n)$ is

$$f(p,theta,n) = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} (theta^{k(n-k)})$$

$$x = 0,1,2,3,...n$$

$$n = 1,2,3,...$$

$$k = 0,1,2,...,n$$

$$0 < p < 1$$

$$0 < theta$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dMultiBin` gives a list format consisting

`pdf` probability function values in vector form.

`mean` mean of Multiplicative Binomial Distribution.

`var` variance of Multiplicative Binomial Distribution.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). "The use of a correlated binomial model for the analysis of certain toxicological experiments." *Biometrics*, 69–76. Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Multiplicative binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dMultiBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dMultiBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],pch=16)
}

dMultiBin(0:10,10,.58,10.022)$pdf   #extracting the pdf values
dMultiBin(0:10,10,.58,10.022)$mean   #extracting the mean
dMultiBin(0:10,10,.58,10.022)$var   #extracting the variance


#plotting random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Multiplicative binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pMultiBin(0:10,10,a[i],1+b[i]),col = col[i],lwd=2.85)
points(0:10,pMultiBin(0:10,10,a[i],1+b[i]),col = col[i],pch=16)
}

pMultiBin(0:10,10,.58,10.022)     #acquiring the cumulative probability values
```

dTRI *Triangular Distribution Bounded Between [0,1]*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Triangular Distribution bounded between [0,1].

Usage

```
dTRI(p,mode)
```

Arguments

p	vector of probabilities.
mode	single value for mode.

Details

Setting $min = 0$ and $max = 1$ $mode = c$ in the Triangular distribution a unit bounded Triangular distribution can be obtained. The probability density function and cumulative density function of a unit bounded Triangular distribution with random variable P are given by

$$g_P(p) = \frac{2p}{c}$$

; $0 \leq p < c$

$$g_P(p) = \frac{2(1-p)}{(1-c)}$$

; $c \leq p \leq 1$

$$G_P(p) = \frac{p^2}{c}$$

; $0 \leq p < c$

$$G_P(p) = 1 - \frac{(1-p)^2}{(1-c)}$$

; $c \leq p \leq 1$

$$0 \leq mode = c \leq 1$$

The mean and the variance are denoted by

$$E[P] = \frac{(a+b+c)}{3} = \frac{(1+c)}{3}$$

$$var[P] = \frac{a^2+b^2+c^2-ab-ac-bc}{18} = \frac{(1+c^2-c)}{18}$$

Moments about zero is denoted as

$$E[P^r] = \frac{2c^{r+2}}{c(r+2)} + \frac{2(1-c^{r+1})}{(1-c)(r+1)} + \frac{2(c^{r+2}-1)}{(1-c)(r+2)}$$

$r = 1, 2, 3, ...$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dTRI` gives a list format consisting

`pdf` probability density values in vector form.

`mean` mean of the unit bounded Triangular distribution.

`variance` variance of the unit bounded Triangular distribution

References

Horsnell G (1957). "Economical acceptance sampling schemes." *Journal of the Royal Statistical Society. Series A (General)*, **120**(2), 148–201. Johnson NL, Kotz S, Balakrishnan N (1995). *Continuous univariate distributions, volume 2*, volume 289. John wiley and sons. Karlis D, Xekalaki E (2008). *The polygonal distribution*. Springer. Okagbue HI, Edeki SO, Opanuga AA, Oguntunde PE, Adeosun ME (2014). "Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution." *British Journal of Mathematics and Computer Science*, **4**(24), 3497.

Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
x <- seq(0.2,0.8,by=0.2)
plot(0,0,main="Probability density graph",xlab="Random variable",
ylab="Probability density values",xlim = c(0,1),ylim = c(0,3))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dTRI(seq(0,1,by=0.01),x[i])$pdf,col = col[i])
}

dTRI(seq(0,1,by=0.05),0.3)$pdf     #extracting the pdf values
dTRI(seq(0,1,by=0.01),0.3)$mean    #extracting the mean
dTRI(seq(0,1,by=0.01),0.3)$var     #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
x <- seq(0.2,0.8,by=0.2)
plot(0,0,main="Cumulative density graph",xlab="Random variable",
ylab="Cumulative density values",xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pTRI(seq(0,1,by=0.01),x[i]),col = col[i])
}
```

```
pTRI(seq(0,1,by=0.05),0.3)      #acquiring the cumulative probability values
mazTRI(1.4,.3)                  #acquiring the moment about zero values
mazTRI(2,.3)-mazTRI(1,.3)^2     #variance for when is mode 0.3

#only the integer value of moments is taken here because moments cannot be decimal
mazTRI(1.9,0.5)
```

---

dTriBin *Triangular Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Triangular Binomial distribution.

Usage

```
dTriBin(x,n,mode)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
mode	single value for mode.

Details

Mixing unit bounded Triangular distribution with Binomial distribution will create Triangular Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{TriBin}(x) = 2\binom{n}{x}(c^{-1}B_c(x+2,n-x+1)+(1-c)^{-1}B(x+1,n-x+2)-(1-c)^{-1}B_c(x+1,n-x+2))$$

$$0 < mode = c < 1$$

$$x = 0, 1, 2, ...n$$

$$n = 1, 2, 3...$$

The mean, variance and over dispersion are denoted as

$$E_{TriiBin}[x] = \frac{n(1+c)}{3}$$

$$Var_{TriBin}[x] = \frac{n(n+3)}{18} - \frac{n(n-3)c(1-c)}{18}$$
$$overdispersion = \frac{(1-c+c^2)}{2(2+c-c^2)}$$

Defined as $B_c(a,b) = \int_0^c t^{a-1}(1-t)^{b-1}\,dt$ is incomplete beta integrals and $B(a,b)$ is the beta function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dTriBin` gives a list format consisting

`pdf` probability function values in vector form.

`mean` mean of the Triangular Binomial Distribution.

`var` variance of the Triangular Binomial Distribution.

`over.dis.para` over dispersion value of the Triangular Binomial Distribution.

References

Horsnell G (1957). "Economical acceptance sampling schemes." *Journal of the Royal Statistical Society. Series A (General)*, **120**(2), 148–201. Karlis D, Xekalaki E (2008). *The polygonal distribution*. Springer. Okagbue HI, Edeki SO, Opanuga AA, Oguntunde PE, Adeosun ME (2014). "Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution." *British Journal of Mathematics and Computer Science*, **4**(24), 3497.

Examples

```
#plotting the random variables and probability values
col <- rainbow(7)
x <- seq(0.1,0.7,by=0.1)
plot(0,0,main="Triangular binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,.3))
for (i in 1:7)
{
lines(0:10,dTriBin(0:10,10,x[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dTriBin(0:10,10,x[i])$pdf,col = col[i],pch=16)
}

dTriBin(0:10,10,.4)$pdf        #extracting the pdf values
dTriBin(0:10,10,.4)$mean       #extracting the mean
dTriBin(0:10,10,.4)$var        #extracting the variance
dTriBin(0:10,10,.4)$over.dis.para  #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(7)
x <- seq(0.1,0.7,by=0.1)
plot(0,0,main="Triangular binomial probability function graph",xlab="Binomial random variable",
```

```
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:7)
{
lines(0:10,pTriBin(0:10,10,x[i]),col = col[i],lwd=2.85)
points(0:10,pTriBin(0:10,10,x[i]),col = col[i],pch=16)
}

pTriBin(0:10,10,.4)    #acquiring the cumulative probability values
```

---

dUNI *Uniform Distribution Bounded Between [0,1]*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Uniform Distribution bounded between [0,1].

Usage

```
dUNI(p)
```

Arguments

p vector of probabilities.

Details

Setting $a = 0$ and $b = 1$ in the Uniform Distribution a unit bounded Uniform Distribution can be obtained. The probability density function and cumulative density function of a unit bounded Uniform Distribution with random variable P are given by

$$g_P(p) = 1$$

$0 \leq p \leq 1$

$$G_P(p) = p$$

$0 \leq p \leq 1$

The mean and the variance are denoted as

$$E[P] = \frac{1}{a+b} = 0.5$$

$$var[P] = \frac{(b-a)^2}{12} = 0.0833$$

Moments about zero is denoted as

$$E[P^r] = \frac{e^{rb} - e^{ra}}{r(b-a)} = \frac{e^r - 1}{r}$$

$r = 1, 2, 3, ...$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of dUNI gives a list format consisting

pdf probability density values in vector form.

mean mean of unit bounded uniform distribution.

var variance of unit bounded uniform distribution.

References

Horsnell G (1957). "Economical acceptance sampling schemes." *Journal of the Royal Statistical Society. Series A (General)*, **120**(2), 148–201. Johnson NL, Kotz S, Balakrishnan N (1995). *Continuous univariate distributions, volume 2*, volume 289. John wiley and sons.

See Also

Uniform

or

https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Uniform.html

Examples

```
#plotting the random variables and probability values
plot(seq(0,1,by=0.01),dUNI(seq(0,1,by=0.01))$pdf,type = "l",main="Probability density graph",
xlab="Random variable",ylab="Probability density values")

dUNI(seq(0,1,by=0.05))$pdf     #extract the pdf values
dUNI(seq(0,1,by=0.01))$mean    #extract the mean
dUNI(seq(0,1,by=0.01))$var     #extract the variance

#plotting the random variables and cumulative probability values
plot(seq(0,1,by=0.01),pUNI(seq(0,1,by=0.01)),type = "l",main="Cumulative density graph",
xlab="Random variable",ylab="Cumulative density values")

pUNI(seq(0,1,by=0.05))     #acquiring the cumulative probability values

mazUNI(c(1,2,3))    #acquiring the moment about zero values

#only the integer value of moments is taken here because moments cannot be decimal
mazUNI(1.9)
```

---

dUniBin *Uniform Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Uniform Binomial Distribution.

Usage

```
dUniBin(x,n)
```

Arguments

`x`	vector of binomial random variables.
`n`	single value for no of binomial trials.

Details

Mixing unit bounded uniform distribution with binomial distribution will create the Uniform Binomial Distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values

$$P_{UniBin}(x) = \frac{1}{n+1}$$

$$n = 1, 2, ...$$

$$x = 0, 1, 2, ...n$$

The mean, variance and over dispersion are denoted as

$$E_{UniBin}[X] = \frac{n}{2}$$

$$Var_{UniBin}[X] = \frac{n(n+2)}{12}$$

$$overdispersion = \frac{1}{3}$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `dUniBin` gives a list format consisting

`pdf` probability function values in vector form.

`mean` mean of the Uniform Binomial Distribution.

`var` variance of the Uniform Binomial Distribution.

`ove.dis.para` over dispersion value of Uniform Binomial Distribution.

References

Horsnell G (1957). "Economical acceptance sampling schemes." *Journal of the Royal Statistical Society. Series A (General)*, **120**(2), 148–201. Okagbue HI, Edeki SO, Opanuga AA, Oguntunde PE, Adeosun ME (2014). "Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution." *British Journal of Mathematics and Computer Science*, **4**(24), 3497.

Examples

```
#plotting the binomial random variables and probability values
plot(0:10,dUniBin(0:10,10)$pdf,type="l",main="Uniform binomial probability function graph",
xlab=" Binomial random variable",ylab="Probability function values")
points(0:10,dUniBin(0:10,10)$pdf)

dUniBin(0:300,300)$pdf  #extracting the pdf values
dUniBin(0:10,10)$mean   #extracting the mean
dUniBin(0:10,10)$var    #extracting the variance
dUniBin(0:10,10)$over.dis.para  #extracting the over dispersion

#plotting the binomial random variables and cumulative probability values
plot(0:10,pUniBin(0:10,10),type="l",main="Cumulative probability function graph",
xlab=" Binomial random variable",ylab="Cumulative probability function values")
points(0:10,pUniBin(0:10,10))

pUniBin(0:15,15)       #acquiring the cumulative probability values
```

Epidemic_Cold *Family Epidemics*

Description

In this investigation, families of the same size, two parents and three children, living in different circumstances of domestic overcrowding were visited at fortnightly intervals. The date of onset and the clinical nature of upper respiratory infectious experienced by each member of the family were charted on a time scale marked off in days. Family epidemics of acute coryza-or common colds-were thus available for analysis.

Usage

```
Epidemic_Cold
```

Format

A data frame with 6 columns and 5 rows

`Cases` No of Further Cases

`Families` No of Families

`Father` Father with Status of Introducing Cases

`Mother` Mother with Status of Introducing Cases

`SChild` School Child with Status of Introducing Cases

`PSChild` Pre-School Child with Status of Introducing Cases

Details

By inspection of the epidemic time charts, it was possible to identify new or primary introductions of illness into the household by the onset of a cold after a lapse of 10 days since the last such case in the same home. Two such cases occurring on the same or succeeding days were classified as multiple primaries. Thereafter, the links in the epidemic chain of spread were defined by an interval of one day or more between successive cases in the same family. These family epidemics could then be described thus 1-2-1, 1-1-1-0, 2-1-0, etc. It must be emphasized that although this method of classification is somewhat arbitrary, it was completed before the corresponding theoretical distributions were worked out and the interval chosen agrees with the distribution of presumptive incubation periods of the common cold seen in field surveys (e.g. Badger, Dingle, Feller, Hodges, Jordan, and Rammelkamp, 1953).

Source

Extracted from

Heasman, M. A. and Reid, D. D. (1961). "Theory and observation in family epidemics of the common cold." Br. J. pleu. SOC. Med., 15, 12-16.

Examples

```
Epidemic_Cold$Cases
sum(Epidemic_Cold$SChild)
```

EstMGFBetaBin — *Estimating the shape parameters a and b for Beta-Binomial Distribution*

Description

The functions will estimate the shape parameters using the maximum log likelihood method and moment generating function method for the Beta-Binomial distribution when the binomial random variables and corresponding frequencies are given.

Usage

```
EstMGFBetaBin(x,freq)
```

Arguments

`x`	vector of binomial random variables.
`freq`	vector of frequencies.

Details

$$a, b > 0$$

$$x = 0, 1, 2, ...$$

$$freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `EstMGFBetaBin` will produce the class `mgf` format consisting

`a` shape parameter of beta distribution representing for alpha

`b` shape parameter of beta distribution representing for beta

`min` Negative loglikelihood value

`AIC` AIC value

`call` the inputs for the function

Methods `print`, `summary`, `coef` and `AIC` can be used to extract specific outputs.

References

Young-Xu Y, Chan KA (2008). "Pooling overdispersed binomial data to estimate event rate." *BMC medical research methodology*, **8**, 1–12. Trenkler G (1996). "Continuous univariate distributions." *Computational Statistics and Data Analysis*, **21**(1), 119–119. HUGHES G, MADDEN L (1993). "Using the beta-binomial distribution to describe aggegated patterns of disease incidence." *Phytopathology*, **83**(7), 759–763.

See Also

`mle2`

Examples

```
No.D.D <- 0:7        #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
estimate <- EstMLEBetaBin(No.D.D,Obs.fre.1,a=0.1,b=0.1)

bbmle::coef(estimate)   #extracting the parameters

#estimating the parameters using moment generating function methods
results <- EstMGFBetaBin(No.D.D,Obs.fre.1)

# extract the estimated parameters and summary
coef(results)
summary(results)
```

```
AIC(results) #show the AIC value
```

`EstMLEAddBin` *Estimating the probability of success and alpha for Additive Binomial Distribution*

Description

The function will estimate the probability of success and alpha using the maximum log likelihood method for the Additive Binomial distribution when the binomial random variables and corresponding frequencies are given.

Usage

```
EstMLEAddBin(x,freq)
```

Arguments

`x`	vector of binomial random variables.
`freq`	vector of frequencies.

Details

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `EstMLEAddBin` will produce the class `mlAB` and `ml` with a list consisting

`min` Negative Log Likelihood value.

`p` estimated probability of success.

`alpha` estimated alpha parameter.

`AIC` AIC value.

`call` the inputs for the function.

Methods `print`, `summary`, `coef` and `AIC` can be used to extract specific outputs.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). "The use of a correlated binomial model for the analysis of certain toxicological experiments." *Biometrics*, 69–76. Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506. Morel JG, Neerchal NK (2012). *Overdispersion models in SAS*. SAS Publishing.

Examples

```
No.D.D <- 0:7         #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)     #assigning the corresponding frequencies

## Not run:
#estimating the probability value and alpha value
results <- EstMLEAddBin(No.D.D,Obs.fre.1)

#printing the summary of results
summary(results)

#extracting the estimated parameters
coef(results)

## End(Not run)
```

`EstMLEBetaBin` *Estimating the shape parameters a and b for Beta-Binomial Distribution*

Description

The functions will estimate the shape parameters using the maximum log likelihood method and moment generating function method for the Beta-Binomial distribution when the binomial random variables and corresponding frequencies are given.

Usage

```
EstMLEBetaBin(x,freq,a,b,...)
```

Arguments

`x`	vector of binomial random variables.
`freq`	vector of frequencies.
`a`	single value for shape parameter alpha representing as a.
`b`	single value for shape parameter beta representing as b.
`...`	mle2 function inputs except data and estimating parameter.

Details

$$a, b > 0$$

$$x = 0, 1, 2, ...$$

$$freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

`EstMLEBetaBin` here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of mle2.

References

Young-Xu Y, Chan KA (2008). "Pooling overdispersed binomial data to estimate event rate." *BMC medical research methodology*, **8**, 1–12. Trenkler G (1996). "Continuous univariate distributions." *Computational Statistics and Data Analysis*, **21**(1), 119–119. HUGHES G, MADDEN L (1993). "Using the beta-binomial distribution to describe aggegated patterns of disease incidence." *Phytopathology*, **83**(7), 759–763.

See Also

`mle2`

Examples

```
No.D.D <- 0:7        #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
estimate <- EstMLEBetaBin(No.D.D,Obs.fre.1,a=0.1,b=0.1)

bbmle::coef(estimate)   #extracting the parameters

#estimating the parameters using moment generating function methods
EstMGFBetaBin(No.D.D,Obs.fre.1)
```

`EstMLEBetaCorrBin` *Estimating the covariance, alpha and beta parameter values for Beta-Correlated Binomial Distribution*

Description

The function will estimate the covariance, alpha and beta parameter values using the maximum log likelihood method for the Beta-Correlated Binomial distribution when the binomial random variables and corresponding frequencies are given.

Usage

```
EstMLEBetaCorrBin(x,freq,cov,a,b,...)
```

Arguments

`x`	vector of binomial random variables.
`freq`	vector of frequencies.
`cov`	single value for covariance.
`a`	single value for alpha parameter.
`b`	single value for beta parameter.
`...`	mle2 function inputs except data and estimating parameter.

Details

$$x = 0, 1, 2, ...$$

$$freq \geq 0$$

$$-\infty < cov < +\infty$$

$$0 < a, b$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

`EstMLEBetaCorrBin` here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of mle2.

References

Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506.

See Also

mle2

Examples

```
No.D.D <- 0:7               #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)     #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEBetaCorrBin(x=No.D.D,freq=Obs.fre.1,cov=0.0050,a=10,b=10)

bbmle::coef(parameters)           #extracting the parameters
```

EstMLECOMPBin — *Estimating the probability of success and v parameter for COM Poisson Binomial Distribution*

Description

The function will estimate the probability of success and v parameter using the maximum log likelihood method for the COM Poisson Binomial distribution when the binomial random variables and corresponding frequencies are given.

Usage

```
EstMLECOMPBin(x,freq,p,v,...)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
p	single value for probability of success.
v	single value for v.
...	mle2 function inputs except data and estimating parameter.

Details

$$x = 0, 1, 2, ...$$
$$freq \geq 0$$
$$0 < p < 1$$
$$-\infty < v < +\infty$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

`EstMLECOMPBin` here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of mle2.

References

Borges P, Rodrigues J, Balakrishnan N, Bazan J (2014). "A COM–Poisson type generalization of the binomial distribution and its properties and applications." *Statistics and Probability Letters*, **87**, 158–166.

Examples

```
No.D.D <- 0:7               #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)     #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLECOMPBin(x=No.D.D,freq=Obs.fre.1,p=0.5,v=0.1)

bbmle::coef(parameters)          #extracting the parameters
```

`EstMLECorrBin` *Estimating the probability of success and correlation for Correlated Binomial Distribution*

Description

The function will estimate the probability of success and correlation using the maximum log likelihood method for the Correlated Binomial distribution when the binomial random variables and corresponding frequencies are given.

Usage

```
EstMLECorrBin(x,freq,p,cov,...)
```

Arguments

`x`	vector of binomial random variables.
`freq`	vector of frequencies.
`p`	single value for probability of success.
`cov`	single value for covariance.
`...`	mle2 function inputs except data and estimating parameter.

Details

$$x = 0, 1, 2, ...$$

$$freq \geq 0$$

$$0 < p < 1$$

$$-\infty < cov < +\infty$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

`EstMLECorrBin` here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of mle2.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). "The use of a correlated binomial model for the analysis of certain toxicological experiments." *Biometrics*, 69–76. Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506. Morel JG, Neerchal NK (2012). *Overdispersion models in SAS*. SAS Publishing.

See Also

`mle2`

Examples

```
No.D.D <- 0:7               #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)     #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLECorrBin(x=No.D.D,freq=Obs.fre.1,p=0.5,cov=0.0050)

bbmle::coef(parameters)          #extracting the parameters
```

`EstMLEGammaBin` *Estimating the shape parameters c and l for Gamma Binomial distribution*

Description

The function will estimate the shape parameters using the maximum log likelihood method for the Gamma Binomial distribution when the binomial random variables and corresponding frequencies are given.

Usage

```
EstMLEGammaBin(x,freq,c,l,...)
```

Arguments

`x`	vector of binomial random variables.
`freq`	vector of frequencies.
`c`	single value for shape parameter c.
`l`	single value for shape parameter l.
`...`	mle2 function inputs except data and estimating parameter.

Details

$$0 < c, l$$

$$x = 0, 1, 2, ...$$

$$freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

`EstMLEGammaBin` here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of mle2.

References

Grassia A (1977). "On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions." *Australian Journal of Statistics*, **19**(2), 108–114.

Examples

```
No.D.D <- 0:7                  #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEGammaBin(x=No.D.D,freq=Obs.fre.1,c=0.1,l=0.1)

bbmle::coef(parameters)        #extracting the parameters
```

`EstMLEGHGBB` *Estimating the shape parameters a,b and c for Gaussian Hypergeometric Generalized Beta Binomial Distribution*

Description

The function will estimate the shape parameters using the maximum log likelihood method for the Gaussian Hypergeometric Generalized Beta Binomial distribution when the binomial random variables and corresponding frequencies are given.

Usage

```
EstMLEGHGBB(x,freq,a,b,c,...)
```

Arguments

`x`	vector of binomial random variables.
`freq`	vector of frequencies.
`a`	single value for shape parameter alpha representing a.
`b`	single value for shape parameter beta representing b.
`c`	single value for shape parameter lambda representing c.
`...`	mle2 function inputs except data and estimating parameter.

Details

$$0 < a, b, c$$
$$x = 0, 1, 2, ...$$
$$freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

`EstMLEGHGBB` here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of mle2.

References

Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ, Olmo-Jimenez MJ (2007). "A generalization of the beta–binomial distribution." *Journal of the Royal Statistical Society Series C: Applied Statistics*, **56**(1), 51–61. Pearson JW (2009). *Computation of hypergeometric functions*. Ph.D. thesis, University of Oxford.

See Also

hypergeo_powerseries

——————-

mle2

Examples

```
No.D.D <- 0:7           #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)     #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEGHGBB(No.D.D,Obs.fre.1,a=0.1,b=0.2,c=0.5)

bbmle::coef(parameters)   #extracting the parameters
```

EstMLEGrassiaIIBin — *Estimating the shape parameters a and b for Grassia II Binomial distribution*

Description

The function will estimate the shape parameters using the maximum log likelihood method for the Grassia II Binomial distribution when the binomial random variables and corresponding frequencies are given.

Usage

```
EstMLEGrassiaIIBin(x,freq,a,b,...)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter a.
b	single value for shape parameter b.
...	mle2 function inputs except data and estimating parameter.

Details

$$0 < a, b$$

$$x = 0, 1, 2, ...$$

$$freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

`EstMLEGrassiaIIBin` here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of mle2.

References

Grassia A (1977). "On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions." *Australian Journal of Statistics*, **19**(2), 108–114.

Examples

```
No.D.D <- 0:7                   #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEGrassiaIIBin(x=No.D.D,freq=Obs.fre.1,a=0.1,b=0.1)

bbmle::coef(parameters)         #extracting the parameters
```

EstMLEKumBin — *Estimating the shape parameters a and b and iterations for Kumaraswamy Binomial Distribution*

Description

The function will estimate the shape parameters using the maximum log likelihood method for the Kumaraswamy Binomial distribution when the binomial random variables and corresponding frequencies are given

Usage

```
EstMLEKumBin(x,freq,a,b,it,...)
```

Arguments

`x`	vector of binomial random variables.
`freq`	vector of frequencies.
`a`	single value for shape parameter alpha representing as a.
`b`	single value for shape parameter beta representing as b.
`it`	number of iterations to converge as a proper probability function replacing infinity.
`...`	mle2 function inputs except data and estimating parameter.

Details

$$0 < a, b$$

$$x = 0, 1, 2, ...$$

$$freq \geq 0$$

$$it > 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

`EstMLEKumBin` here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of mle2.

References

Xiaohu L, Yanyan H, Xueyan Z (2011). "The Kumaraswamy binomial distribution." *Chinese Journal of Applied Probability and Statistics*, **27**(5), 511–521.

See Also

`mle2`

Examples

```
No.D.D <- 0:7     #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

## Not run:
#estimating the parameters using maximum log likelihood value and assigning it
parameters1 <- EstMLEKumBin(x=No.D.D,freq=Obs.fre.1,a=10.1,b=1.1,it=10000)

bbmle::coef(parameters1)   #extracting the parameters

## End(Not run)
```

`EstMLELMBin` *Estimating the probability of success and theta for Lovinson Multiplicative Binomial Distribution*

Description

The function will estimate the probability of success and phi parameter using the maximum log likelihood method for the Lovinson Multiplicative Binomial distribution when the binomial random variables and corresponding frequencies are given.

Usage

```
EstMLELMBin(x,freq,p,phi,...)
```

Arguments

`x`	vector of binomial random variables.
`freq`	vector of frequencies.
`p`	single value for probability of success.
`phi`	single value for phi parameter.
`...`	mle2 function inputs except data and estimating parameter.

Details

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$0 < phi$$

Value

`EstMLELMBin` here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of mle2.

References

Elamir EA (2013). "Multiplicative-Binomial Distribution: Some Results on Characterization, Inference and Random Data Generation." *Journal of Statistical Theory and Applications*, **12**(1), 92–105.

See Also

`mle2`

Examples

```
No.D.D <- 0:7         #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLELMBin(x=No.D.D,freq=Obs.fre.1,p=0.5,phi=15)

bbmle::coef(parameters)           #extracting the parameters
```

EstMLEMcGBB — *Estimating the shape parameters a,b and c for McDonald Generalized Beta Binomial distribution*

Description

The function will estimate the shape parameters using the maximum log likelihood method for the McDonald Generalized Beta Binomial distribution when the binomial random variables and corresponding frequencies are given.

Usage

```
EstMLEMcGBB(x,freq,a,b,c,...)
```

Arguments

`x`	vector of binomial random variables.
`freq`	vector of frequencies.
`a`	single value for shape parameter alpha representing as a.
`b`	single value for shape parameter beta representing as b.
`c`	single value for shape parameter gamma representing as c.
`...`	mle2 function inputs except data and estimating parameter.

Details

$$0 < a, b, c$$
$$x = 0, 1, 2, ...$$
$$freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

`EstMLEMcGBB` here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of mle2.

References

Manoj C, Wijekoon P, Yapa RD (2013). "The McDonald generalized beta-binomial distribution: A new binomial mixture distribution and simulation based comparison with its nested distributions in handling overdispersion." *International journal of statistics and probability*, **2**(2), 24. Janiffer NM, Islam A, Luke O, others (2014). "Estimating Equations for Estimation of Mcdonald Generalized Beta—Binomial Parameters." *Open Journal of Statistics*, **4**(09), 702. Roozegar R, Tahmasebi S, Jafari AA (2017). "The McDonald Gompertz distribution: properties and applications." *Communications in Statistics-Simulation and Computation*, **46**(5), 3341–3355.

See Also

mle2

Examples

```
No.D.D <- 0:7                   #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

## Not run:
#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEMcGBB(x=No.D.D,freq=Obs.fre.1,a=0.1,b=0.1,c=0.2)

bbmle::coef(parameters)         #extracting the parameters

## End(Not run)
```

EstMLEMultiBin   *Estimating the probability of success and theta for Multiplicative Binomial Distribution*

Description

The function will estimate the probability of success and theta parameter using the maximum log likelihood method for the Multiplicative Binomial distribution when the binomial random variables and corresponding frequencies are given.

Usage

```
EstMLEMultiBin(x,freq,p,theta,...)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
p	single value for probability of success.
theta	single value for theta parameter.
...	mle2 function inputs except data and estimating parameter.

Details

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$0 < theta$$

Value

`EstMLEMultiBin` here is used as a wrapper for the `mle2` function of **bbmle** package therefore output is of class of mle2.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). "The use of a correlated binomial model for the analysis of certain toxicological experiments." *Biometrics*, 69–76. Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506.

See Also

`mle2`

Examples

```
No.D.D <- 0:7         #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEMultiBin(x=No.D.D,freq=Obs.fre.1,p=0.5,theta=15)

bbmle::coef(parameters)           #extracting the parameters
```

EstMLETriBin — *Estimating the mode value for Triangular Binomial Distribution*

Description

The function will estimate the mode value using the maximum log likelihood method for the Triangular Binomial Distribution when the binomial random variables and corresponding frequencies are given.

Usage

```
EstMLETriBin(x,freq)
```

Arguments

`x`	vector of binomial random variables.
`freq`	vector of frequencies.

Details

$$0 < mode = c < 1$$

$$x = 0, 1, 2, ...$$

$$freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `EstMLETriBin` will produce the classes of `ml` and `mlTB` format consisting

`min` Negative log likelihood value.

`mode` Estimated mode value.

`AIC` AIC value.

`call` the inputs for the function.

Methods `print`, `summary`, `coef` and `AIC` can be used to extract specific outputs.

References

Horsnell G (1957). "Economical acceptance sampling schemes." *Journal of the Royal Statistical Society. Series A (General)*, **120**(2), 148–201. Karlis D, Xekalaki E (2008). *The polygonal distribution*. Springer. Okagbue HI, Edeki SO, Opanuga AA, Oguntunde PE, Adeosun ME (2014). "Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution." *British Journal of Mathematics and Computer Science*, **4**(24), 3497.

Examples

```
No.D.D <- 0:7   #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

## Not run:
#estimating the mode value and extracting the mode value
results <- EstMLETriBin(No.D.D,Obs.fre.1)

# extract the mode value and summary
coef(results)
summary(results)

AIC(results)  #show the AIC value
```

```
## End(Not run)
```

`Exam_data` *Exam Data*

Description

In an examination, there were 9 questions set on a particular topic. Each question is marked out of a total of 20 and in assessing the final class of a candidate, particular attention is paid to the total number of questions for which he has an "alpha", i.e., at least 15 out of 20, as well as his total number of marks. His number of alpha's is a rough indication of the "quality" of his exam performance. Thus, the distribution of alpha's over the candidates is of interest. There were 209 candidates attempting questions from this section of 9 questions and a total of 326 alpha's was awarded. So we treat 9 as the "litter size", and the dichotomous response is whether or not he got an alpha on the question.

Usage

```
Exam_data
```

Format

A data frame with 2 columns and 10 rows

`No.of.alpha` No of Alphas

`fre` Observed frequencies

Source

Extracted from

Paul, S.R., 1985. A three-parameter generalization of the binomial distribution. Communications in Statistics - Theory and Methods, 14(6), pp.1497-1506.

Available at: doi:10.1080/03610928508828990

Examples

```
Exam_data$No.of.alpha             #extracting the binomial random variables
sum(Exam_data$fre)                #summing all the frequencies
```

`fitAddBin` *Fitting the Additive Binomial Distribution when binomial random variable, frequency, probability of success and alpha are given*

Description

The function will fit the Additive Binomial distribution when random variables, corresponding frequencies, probability of success and alpha are given. It will provide the expected frequencies, chi-squared test statistics value, p value, and degree of freedom value so that it can be seen if this distribution fits the data.

Usage

```
fitAddBin(x,obs.freq,p,alpha)
```

Arguments

`x`	vector of binomial random variables.
`obs.freq`	vector of frequencies.
`p`	single value for probability of success.
`alpha`	single value for alpha.

Details

$$obs.freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$-1 < alpha < 1$$

Value

The output of `fitAddBin` gives the class format `fitAB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`fitAB` fitted probability values of `dAddBin`.

`NegLL` Negative Log Likelihood value.

`p` estimated probability value.

`alpha` estimated alpha parameter value.

`AIC` AIC value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fitted` can be used to extract specific outputs.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). "The use of a correlated binomial model for the analysis of certain toxicological experiments." *Biometrics*, 69–76. Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506. Morel JG, Neerchal NK (2012). *Overdispersion models in SAS*. SAS Publishing.

Examples

```
No.D.D <- 0:7         #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)        #assigning the corresponding the frequencies

## Not run:
#assigning the estimated probability value
paddbin <- EstMLEAddBin(No.D.D,Obs.fre.1)$p

#assigning the estimated alpha value
alphaaddbin <- EstMLEAddBin(No.D.D,Obs.fre.1)$alpha

#fitting when the random variable,frequencies,probability and alpha are given
results <- fitAddBin(No.D.D,Obs.fre.1,paddbin,alphaaddbin)
results

#extracting the AIC value
AIC(results)

#extract fitted values
fitted(results)

## End(Not run)
```

`fitBetaBin` *Fitting the Beta-Binomial Distribution when binomial random variable, frequency and shape parameters a and b are given*

Description

The function will fit the Beta-Binomial distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

Usage

```
fitBetaBin(x,obs.freq,a,b)
```

Arguments

`x`	vector of binomial random variables.
`obs.freq`	vector of frequencies.
`a`	single value for shape parameter alpha representing as a.
`b`	single value for shape parameter beta representing as b.

Details

$$0 < a, b$$
$$x = 0, 1, 2, ..., n$$
$$obs.freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `fitBetaBin` gives the class format `fitBB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`fitBB` fitted values of `dBetaBin`.

`NegLL` Negative Log Likelihood value.

`a` estimated value for alpha parameter as a.

`b` estimated value for alpha parameter as b.

`AIC` AIC value.

`over.dis.para` over dispersion value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fitted` can be used to extract specific outputs.

References

Young-Xu Y, Chan KA (2008). "Pooling overdispersed binomial data to estimate event rate." *BMC medical research methodology*, **8**, 1–12. Trenkler G (1996). "Continuous univariate distributions." *Computational Statistics and Data Analysis*, **21**(1), 119–119. HUGHES G, MADDEN L (1993). "Using the beta-binomial distribution to describe aggegated patterns of disease incidence." *Phytopathology*, **83**(7), 759–763.

See Also

mle2

Examples

```
No.D.D <- 0:7    #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEBetaBin(No.D.D,Obs.fre.1,0.1,0.1)

bbmle::coef(parameters)   #extracting the parameters a and b
aBetaBin <- bbmle::coef(parameters)[1]  #assigning the parameter a
bBetaBin <- bbmle::coef(parameters)[2]  #assigning the parameter b

#fitting when the random variable,frequencies,shape parameter values are given.
fitBetaBin(No.D.D,Obs.fre.1,aBetaBin,bBetaBin)

#estimating the parameters using moment generating function methods
results <- EstMGFBetaBin(No.D.D,Obs.fre.1)
results

aBetaBin1 <- results$a  #assigning the estimated a
bBetaBin1 <- results$b  #assigning the estimated b

#fitting when the random variable,frequencies,shape parameter values are given.
BB <- fitBetaBin(No.D.D,Obs.fre.1,aBetaBin1,bBetaBin1)

#extracting the expected frequencies
fitted(BB)

#extracting the residuals
residuals(BB)
```

---

fitBetaCorrBin — *Fitting the Beta-Correlated Binomial Distribution when binomial random variable, frequency, covariance, alpha and beta parameters are given*

Description

The function will fit the Beta-Correlated Binomial Distribution when random variables, corresponding frequencies, covariance, alpha and beta parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, and degree of freedom so that it can be seen if this distribution fits the data.

Usage

```
fitBetaCorrBin(x,obs.freq,cov,a,b)
```

Arguments

x	vector of binomial random variables.
obs.freq	vector of frequencies.
cov	single value for covariance.
a	single value for alpha parameter.
b	single value for beta parameter.

Details

$$obs.freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$-\infty < cov < +\infty$$

$$0 < a, b$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `fitBetaCorrBin` gives the class format `fitBCB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic

`corr` Correlation value.

`fitBCB` fitted probability values of `dBetaCorrBin`.

`NegLL` Negative Log Likelihood value.

`a` estimated shape parameter value a.

`b` estimated shape parameter value b.

`cov` estimated covariance value.

`AIC` AIC value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fitted` can be used to extract specific outputs.

References

Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506.

Examples

```
No.D.D <- 0:7                    #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEBetaCorrBin(x=No.D.D,freq=Obs.fre.1,cov=0.0050,a=10,b=10)

covBetaCorrBin <- bbmle::coef(parameters)[1]
aBetaCorrBin <- bbmle::coef(parameters)[2]
bBetaCorrBin <- bbmle::coef(parameters)[3]

#fitting when the random variable,frequencies,covariance, a and b are given
results <- fitBetaCorrBin(No.D.D,Obs.fre.1,covBetaCorrBin,aBetaCorrBin,bBetaCorrBin)
results

#extract AIC value
AIC(results)

#extract fitted values
fitted(results)
```

`fitBin` *Fitting the Binomial Distribution when binomial random variable, frequency and probability value are given*

Description

The function will fit the Binomial distribution when random variables, corresponding frequencies and probability value are given. It will provide the expected frequencies, chi-squared test statistics value, p value and degree of freedom so that it can be seen if this distribution fits the data.

Usage

```
fitBin(x,obs.freq,p=0)
```

Arguments

`x`	vector of binomial random variables.
`obs.freq`	vector of frequencies.
`p`	single value for probability or zero to estimate p.

Details

$$x = 0, 1, 2, ...$$

$$0 < p <= 1$$

$$obs.freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `fitBin` gives the class format `fitB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics value.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`fitB` fitted probability values of `dbinom`.

`phat` estimated probability value.

`call` the inputs of the function.

Examples

```
No.D.D <- 0:7      #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

#fitting when the random variable,frequencies are given.
fitBin(No.D.D,Obs.fre.1)
```

`fitCOMPBin` *Fitting the COM Poisson Binomial Distribution when binomial random variable, frequency, probability of success and v parameter are given*

Description

The function will fit the COM Poisson Binomial Distribution when random variables, corresponding frequencies, probability of success and v parameter are given. It will provide the expected frequencies, chi-squared test statistics value, p value, and degree of freedom so that it can be seen if this distribution fits the data.

Usage

```
fitCOMPBin(x,obs.freq,p,v)
```

Arguments

`x`	vector of binomial random variables.
`obs.freq`	vector of frequencies.
`p`	single value for probability of success.
`v`	single value for v.

Details

$$obs.freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$-\infty < v < +\infty$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `fitCOMPBin` gives the class format `fitCPB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`fitCPB` fitted probability values of `dCOMPBin`.

`NegLL` Negative Log Likelihood value.

`p` estimated probability value.

`v` estimated v parameter value.

`AIC` AIC value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fitted` can be used to extract specific outputs.

References

Borges P, Rodrigues J, Balakrishnan N, Bazan J (2014). "A COM–Poisson type generalization of the binomial distribution and its properties and applications." *Statistics and Probability Letters*, **87**, 158–166.

Examples

```
No.D.D <- 0:7                    #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLECOMPBin(x=No.D.D,freq=Obs.fre.1,p=0.5,v=0.050)

pCOMPBin <- bbmle::coef(parameters)[1]
vCOMPBin <- bbmle::coef(parameters)[2]

#fitting when the random variable,frequencies,probability and v parameter are given
results <- fitCOMPBin(No.D.D,Obs.fre.1,pCOMPBin,vCOMPBin)
results

#extracting the AIC value
AIC(results)

#extract fitted values
fitted(results)
```

---

fitCorrBin — *Fitting the Correlated Binomial Distribution when binomial random variable, frequency, probability of success and covariance are given*

Description

The function will fit the Correlated Binomial Distribution when random variables, corresponding frequencies, probability of success and covariance are given. It will provide the expected frequencies, chi-squared test statistics value, p value, and degree of freedom so that it can be seen if this distribution fits the data.

Usage

```
fitCorrBin(x,obs.freq,p,cov)
```

Arguments

x	vector of binomial random variables.
obs.freq	vector of frequencies.
p	single value for probability of success.
cov	single value for covariance.

Details

$$obs.freq \geq 0$$
$$x = 0, 1, 2, ..$$
$$0 < p < 1$$
$$-\infty < cov < +\infty$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `fitCorrBin` gives the class format `fitCB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`corr` Correlation value.

`fitCB` fitted probability values of `dCorrBin`.

`NegLL` Negative Log Likelihood value.

`AIC` AIC value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fitted` can be used to extract specific outputs.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). “The use of a correlated binomial model for the analysis of certain toxicological experiments.” *Biometrics*, 69–76. Paul SR (1985). “A three-parameter generalization of the binomial distribution.” *History and Philosophy of Logic*, **14**(6), 1497–1506. Morel JG, Neerchal NK (2012). *Overdispersion models in SAS*. SAS Publishing.

Examples

```
No.D.D <- 0:7       #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLECorrBin(x=No.D.D,freq=Obs.fre.1,p=0.5,cov=0.0050)

pCorrBin <- bbmle::coef(parameters)[1]
covCorrBin <- bbmle::coef(parameters)[2]
```

```
#fitting when the random variable,frequencies,probability and covariance are given
results <- fitCorrBin(No.D.D,Obs.fre.1,pCorrBin,covCorrBin)
results

#extracting the AIC value
AIC(results)

#extract fitted values
fitted(results)
```

---

fitGammaBin — *Fitting the Gamma Binomial distribution when binomial random variable, frequency and shape parameters are given*

Description

The function will fit the Gamma Binomial Distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

Usage

```
fitGammaBin(x,obs.freq,c,l)
```

Arguments

`x`	vector of binomial random variables.
`obs.freq`	vector of frequencies.
`c`	single value for shape parameter c.
`l`	single value for shape parameter l.

Details

$$0 < c, l$$

$$x = 0, 1, 2, ...$$

$$obs.freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `fitGammaBin` gives the class format `fitGaB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`fitMB` fitted values of `dGammaBin`.

`NegLL` Negative Log Likelihood value.

`c` estimated value for shape parameter c.

`l` estimated value for shape parameter l.

`AIC` AIC value.

`over.dis.para` over dispersion value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fitted` can be used to extract specific outputs.

References

Grassia A (1977). "On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions." *Australian Journal of Statistics*, **19**(2), 108–114.

Examples

```
No.D.D <- 0:7       #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)         #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEGammaBin(x=No.D.D,freq=Obs.fre.1,c=0.1,l=0.1)

cGBin <- bbmle::coef(parameters)[1]         #assigning the estimated c
lGBin <- bbmle::coef(parameters)[2]         #assigning the estimated l

#fitting when the random variable,frequencies,shape parameter values are given.
results <- fitGammaBin(No.D.D,Obs.fre.1,cGBin,lGBin)
results

#extracting the expected frequencies
fitted(results)

#extracting the residuals
residuals(results)
```

`fitGHGBB` *Fitting the Gaussian Hypergeometric Generalized Beta Binomial Distribution when binomial random variable, frequency and shape parameters a,b and c are given*

Description

The function will fit the Gaussian Hypergeometric Generalized Beta Binomial Distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

Usage

```
fitGHGBB(x,obs.freq,a,b,c)
```

Arguments

`x`	vector of binomial random variables.
`obs.freq`	vector of frequencies.
`a`	single value for shape parameter alpha representing a.
`b`	single value for shape parameter beta representing b.
`c`	single value for shape parameter lambda representing c.

Details

$$0 < a, b, c$$

$$x = 0, 1, 2, ...$$

$$obs.freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `fitGHGBB` gives the class format `fitGB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`fitGB` fitted values of `dGHGBB`.

`NegLL` Negative Loglikelihood value.

`a` estimated value for alpha parameter as a.

`b` estimated value for beta parameter as b.

`c` estimated value for gamma parameter as c.

`AIC` AIC value.

`over.dis.para` over dispersion value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fitted` can be used to extract specific outputs.

References

Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ, Olmo-Jimenez MJ (2007). "A generalization of the beta–binomial distribution." *Journal of the Royal Statistical Society Series C: Applied Statistics*, **56**(1), 51–61. Pearson JW (2009). *Computation of hypergeometric functions*. Ph.D. thesis, University of Oxford.

See Also

`hypergeo_powerseries`

`mle2`

Examples

```
No.D.D <- 0:7        #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)       #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEGHGBB(No.D.D,Obs.fre.1,0.1,20,1.3)

bbmle::coef(parameters)         #extracting the parameters
aGHGBB <- bbmle::coef(parameters)[1]  #assigning the estimated a
bGHGBB <- bbmle::coef(parameters)[2]  #assigning the estimated b
cGHGBB <- bbmle::coef(parameters)[3]  #assigning the estimated c

#fitting when the random variable,frequencies,shape parameter values are given.
results <- fitGHGBB(No.D.D,Obs.fre.1,aGHGBB,bGHGBB,cGHGBB)
results

#extracting the expected frequencies
fitted(results)

#extracting the residuals
residuals(results)
```

`fitGrassiaIIBin` *Fitting the Grassia II Binomial distribution when binomial random variable, frequency and shape parameters are given*

Description

The function will fit the Grassia II Binomial Distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

Usage

```
fitGrassiaIIBin(x,obs.freq,a,b)
```

Arguments

`x`	vector of binomial random variables.
`obs.freq`	vector of frequencies.
`a`	single value for shape parameter a.
`b`	single value for shape parameter b.

Details

$$0 < a, b$$

$$x = 0, 1, 2, ...$$

$$obs.freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `fitGrassiaIIBin` gives the class format `fitGrIIB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`fitGrIIB` fitted values of `dGrassiaIIBin`.

`NegLL` Negative Log Likelihood value.

`a` estimated value for shape parameter a.

`b` estimated value for shape parameter b.

`AIC` AIC value.

`over.dis.para` over dispersion value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fitted` can be used to extract specific outputs.

References

Grassia A (1977). "On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions." *Australian Journal of Statistics*, **19**(2), 108–114.

Examples

```
No.D.D <- 0:7       #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)         #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEGrassiaIIBin(x=No.D.D,freq=Obs.fre.1,a=0.1,b=0.1)

aGIIBin <- bbmle::coef(parameters)[1]         #assigning the estimated a
bGIIBin <- bbmle::coef(parameters)[2]         #assigning the estimated b

#fitting when the random variable,frequencies,shape parameter values are given.
results <- fitGrassiaIIBin(No.D.D,Obs.fre.1,aGIIBin,bGIIBin)
results

#extracting the expected frequencies
fitted(results)

#extracting the residuals
residuals(results)
```

`fitKumBin` *Fitting the Kumaraswamy Binomial Distribution when binomial random variable, frequency and shape parameters a and b, iterations parameter it are given*

Description

The function will fit the Kumaraswamy Binomial distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

Usage

```
fitKumBin(x,obs.freq,a,b,it)
```

Arguments

`x`	vector of binomial random variables.
`obs.freq`	vector of frequencies.
`a`	single value for shape parameter alpha representing a.
`b`	single value for shape parameter beta representing b.
`it`	number of iterations to converge as a proper probability function replacing infinity.

Details

$$0 < a, b$$

$$x = 0, 1, 2, ...n$$

$$obs.freq \geq 0$$

$$it > 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `fitKumBin` gives the class format `fitKB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`fitKB` fitted values of `dKumBin`.

`NegLL` Negative Log Likelihood value.

`a` estimated value for alpha parameter as a.

`b` estimated value for beta parameter as b.

`it` estimated it value for iterations.

`AIC` AIC value.

`over.dis.para` over dispersion value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fiited` can be used to extract specific outputs.

References

Xiaohu L, Yanyan H, Xueyan Z (2011). "The Kumaraswamy binomial distribution." *Chinese Journal of Applied Probability and Statistics*, **27**(5), 511–521.

See Also

mle2

Examples

```
No.D.D <- 0:7   #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

## Not run:
#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEKumBin(x=No.D.D,freq=Obs.fre.1,a=10.1,b=1.1,it=10000)

bbmle::coef(parameters)    #extracting the parameters
aKumBin <- bbmle::coef(parameters)[1] #assigning the estimated a
bKumBin <- bbmle::coef(parameters)[2] #assigning the estimated b
itKumBin <- bbmle::coef(parameters)[3] #assigning the estimated iterations

#fitting when the random variable,frequencies,shape parameter values are given.
results <- fitKumBin(No.D.D,Obs.fre.1,aKumBin,bKumBin,itKumBin*100)
results

#extracting the expected frequencies
fitted(results)

#extracting the residuals
residuals(results)

## End(Not run)
```

---

fitLMBin — *Fitting the Lovinson Multiplicative Binomial Distribution when binomial random variable, frequency, probability of success and theta parameter are given*

Description

The function will fit the Lovinson Multiplicative Binomial distribution when random variables, corresponding frequencies, probability of success and phi parameter are given. It will provide the expected frequencies, chi-squared test statistics value, p value and degree of freedom value so that it can be seen if this distribution fits the data.

Usage

```
fitLMBin(x,obs.freq,p,phi)
```

Arguments

x	vector of binomial random variables.
obs.freq	vector of frequencies.
p	single value for probability of success.
phi	single value for phi parameter.

Details

$$obs.freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$0 < phi$$

Value

The output of `fitLMBin` gives the class format `fitLMB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`fitLMB` fitted probability values of `dLMBin`.

`NegLL` Negative Log Likelihood value.

`p` estimated probability value.

`phi` estimated phi parameter value.

`AIC` AIC value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fitted` can be used to extract specific outputs.

References

Elamir EA (2013). "Multiplicative-Binomial Distribution: Some Results on Characterization, Inference and Random Data Generation." *Journal of Statistical Theory and Applications*, **12**(1), 92–105.

See Also

`mle2`

Examples

```
No.D.D <- 0:7       #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)     #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLELMBin(x=No.D.D,freq=Obs.fre.1,p=0.1,phi=.3)

pLMBin=bbmle::coef(parameters)[1]    #assigning the estimated probability value
phiLMBin <- bbmle::coef(parameters)[2]  #assigning the estimated phi value

#fitting when the random variable,frequencies,probability and phi are given
results <- fitLMBin(No.D.D,Obs.fre.1,pLMBin,phiLMBin)
results

#extracting the AIC value
AIC(results)

#extract fitted values
fitted(results)
```

`fitMcGBB` *Fitting the McDonald Generalized Beta Binomial distribution when binomial random variable, frequency and shape parameters are given*

Description

The function will fit the McDonald Generalized Beta Binomial Distribution when random variables, corresponding frequencies and shape parameters are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

Usage

```
fitMcGBB(x,obs.freq,a,b,c)
```

Arguments

`x`	vector of binomial random variables.
`obs.freq`	vector of frequencies.
`a`	single value for shape parameter alpha representing a.
`b`	single value for shape parameter beta representing b.
`c`	single value for shape parameter gamma representing c.

Details

$$0 < a, b, c$$

$$x = 0, 1, 2, ...$$

$$obs.freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `fitMcGBB` gives the class format `fitMB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`fitMB` fitted values of `dMcGBB`.

`NegLL` Negative Log Likelihood value.

`a` estimated value for alpha parameter as a.

`b` estimated value for beta parameter as b.

`c` estimated value for gamma parameter as c.

`AIC` AIC value.

`over.dis.para` over dispersion value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fitted` can be used to extract specific outputs.

References

Manoj C, Wijekoon P, Yapa RD (2013). "The McDonald generalized beta-binomial distribution: A new binomial mixture distribution and simulation based comparison with its nested distributions in handling overdispersion." *International journal of statistics and probability*, **2**(2), 24. Janiffer NM, Islam A, Luke O, others (2014). "Estimating Equations for Estimation of Mcdonald Generalized Beta—Binomial Parameters." *Open Journal of Statistics*, **4**(09), 702. Roozegar R, Tahmasebi S, Jafari AA (2017). "The McDonald Gompertz distribution: properties and applications." *Communications in Statistics-Simulation and Computation*, **46**(5), 3341–3355.

See Also

`mle2`

Examples

```
No.D.D <- 0:7       #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)         #assigning the corresponding frequencies

## Not run:
#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEMcGBB(x=No.D.D,freq=Obs.fre.1,a=0.1,b=0.1,c=3.2)

aMcGBB <- bbmle::coef(parameters)[1]         #assigning the estimated a
bMcGBB <- bbmle::coef(parameters)[2]         #assigning the estimated b
cMcGBB <- bbmle::coef(parameters)[3]         #assigning the estimated c

#fitting when the random variable,frequencies,shape parameter values are given.
results <- fitMcGBB(No.D.D,Obs.fre.1,aMcGBB,bMcGBB,cMcGBB)
results

#extracting the expected frequencies
fitted(results)

#extracting the residuals
residuals(results)

## End(Not run)
```

`fitMultiBin` *Fitting the Multiplicative Binomial Distribution when binomial random variable, frequency, probability of success and theta parameter are given*

Description

The function will fit the Multiplicative Binomial distribution when random variables, corresponding frequencies, probability of success and theta parameter are given. It will provide the expected frequencies, chi-squared test statistics value, p value and degree of freedom value so that it can be seen if this distribution fits the data.

Usage

```
fitMultiBin(x,obs.freq,p,theta)
```

Arguments

`x`	vector of binomial random variables.
`obs.freq`	vector of frequencies.
`p`	single value for probability of success.
`theta`	single value for theta parameter.

Details

$$obs.freq \geq 0$$
$$x = 0, 1, 2, ..$$
$$0 < p < 1$$
$$0 < theta$$

Value

The output of `fitMultiBin` gives the class format `fitMuB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`fitMuB` fitted probability values of `dMultiBin`.

`NegLL` Negative Log Likelihood value.

`p` estimated probability value.

`theta` estimated theta parameter value.

`AIC` AIC value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fitted` can be used to extract specific outputs.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). “The use of a correlated binomial model for the analysis of certain toxicological experiments.” *Biometrics*, 69–76. Paul SR (1985). “A three-parameter generalization of the binomial distribution.” *History and Philosophy of Logic*, **14**(6), 1497–1506.

See Also

`mle2`

Examples

```
No.D.D <- 0:7       #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)     #assigning the corresponding frequencies

#estimating the parameters using maximum log likelihood value and assigning it
parameters <- EstMLEMultiBin(x=No.D.D,freq=Obs.fre.1,p=0.1,theta=.3)

pMultiBin <- bbmle::coef(parameters)[1]    #assigning the estimated probability value
```

```
thetaMultiBin <- bbmle::coef(parameters)[2]  #assigning the estimated theta value

#fitting when the random variable,frequencies,probability and theta are given
results <- fitMultiBin(No.D.D,Obs.fre.1,pMultiBin,thetaMultiBin)
results

#extracting the AIC value
AIC(results)

#extract fitted values
fitted(results)
```

`fitTriBin` *Fitting the Triangular Binomial Distribution when binomial random variable, frequency and mode value are given*

Description

The function will fit the Triangular Binomial distribution when random variables, corresponding frequencies and mode parameter are given. It will provide the expected frequencies, chi-squared test statistics value, p value, degree of freedom and over dispersion value so that it can be seen if this distribution fits the data.

Usage

```
fitTriBin(x,obs.freq,mode)
```

Arguments

`x`	vector of binomial random variables.
`obs.freq`	vector of frequencies.
`mode`	single value for mode.

Details

$$0 < mode = c < 1$$

$$x = 0, 1, 2, ...$$

$$0 < mode < 1$$

$$obs.freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `fitTriBin` gives the class format `fitTB` and `fit` consisting a list

`bin.ran.var` binomial random variables.

`obs.freq` corresponding observed frequencies.

`exp.freq` corresponding expected frequencies.

`statistic` chi-squared test statistics value.

`df` degree of freedom.

`p.value` probability value by chi-squared test statistic.

`fitTB` fitted probability values of `dTriBin`.

`NegLL` Negative Log Likelihood value.

`mode` estimated mode value.

`AIC` AIC value.

`over.dis.para` over dispersion value.

`call` the inputs of the function.

Methods `summary`, `print`, `AIC`, `residuals` and `fitted` can be used to extract specific outputs.

References

Horsnell G (1957). "Economical acceptance sampling schemes." *Journal of the Royal Statistical Society. Series A (General)*, **120**(2), 148–201. Karlis D, Xekalaki E (2008). *The polygonal distribution*. Springer. Okagbue HI, Edeki SO, Opanuga AA, Oguntunde PE, Adeosun ME (2014). "Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution." *British Journal of Mathematics and Computer Science*, **4**(24), 3497.

Examples

```
No.D.D <- 0:7      #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

modeTriBin <- EstMLETriBin(No.D.D,Obs.fre.1)$mode  #assigning the extracted the mode value

#fitting when the random variable,frequencies,mode value are given.
results <- fitTriBin(No.D.D,Obs.fre.1,modeTriBin)
results

#extract AIC value
AIC(results)

#extract fitted values
fitted(results)
```

GenerateBOD *Generate Overdispersed Binomial Outcome Data*

Description

Using a three step algorithm to generate overdispersed binomial outcome data. When the number of frequencies, binomial random variable, probability of success and overdispersion are given.

Usage

```
GenerateBOD(N,n,pi,rho)
```

Arguments

N	single value for number of total frequencies
n	single value for binomial random variable
pi	single value for probability of success
rho	single value for overdispersion parameter

Details

The generated binomial random variables are overdispersed based on rho for the probability of success pi.

Step 1: Solve the following equation for a given n, pi, rho,

$$phi(z(pi), z(pi), delta) = pi(1 - pi)rho + pi^2,$$

For $delta$ where $phi(z(pi), z(pi), delta)$ is the cumulative distribution function of the standard bivariate normal random variable with correlation coefficient $delta$, and $z(pi)$ denotes the pi^{th} quantile of the standard normal distribution.

Step 2: Generate \$n\$-dimensional multivariate normal random variables, $Z_i = (Z_{i1}, Z_{i2}, ldots, Z_{in})^T$ with mean 0 and constant correlation matrix $Sigma_i$ for $i = 1, 2, \ldots, N$, where the elements of $(Sigma_i)_{lm}$ are $delta$ for $l \neq m$.

Step 3: Now for each $j = 1, 2, \ldots, n$ define $X_{ij} = 1$; if $Z_{ij} < z(\pi)$, or $X_{ij} = 0$; otherwise. Then, it can be showed that the random variable $Y_i = \sum_{j=1}^{n} X_{ij}$ is overdispersed relative to the Binomial distribution.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of GenerateBOD gives a vector of overdispersed binomial random variables

References

Manoj C, Wijekoon P, Yapa RD (2013). "The McDonald generalized beta-binomial distribution: A new binomial mixture distribution and simulation based comparison with its nested distributions in handling overdispersion." *International journal of statistics and probability*, **2**(2), 24.

Examples

```
N <- 500    # Number of observations
n <- 10      # Dimension of multivariate normal random variables
pi <- 0.5   # Probability threshold
rho <- 0.1  # Dispersion parameter

# Generate overdispersed binomial variables
New_overdispersed_data <- GenerateBOD(N, n, pi, rho)
table(New_overdispersed_data)
```

`Male_Children` *Male children data*

Description

The number of male children among the first 12 children of family size 13 in 6115 families taken from the hospital records in the nineteenth century Saxony (Sokal & Rohlf(1994), Lindsey (1995), p. 59). The thirteenth child is ignored to assuage the effect of families non-randomly stopping when a desired gender is reached.

Usage

```
Male_Children
```

Format

A data frame with 2 columns and 13 rows.

`No_of_Males` No of Male children among first 12 children of family size 13

`freq` Observed frequencies for corresponding male children

Source

Extracted from

Borges, P., Rodrigues, J., Balakrishnan, N. and Bazan, J., 2014. A COM-Poisson type generalization of the binomial distribution and its properties and applications. Statistics & Probability Letters, 87, pp.158-166.

Available at: doi:10.1016/j.spl.2014.01.019

Examples

```
Male_Children$No_of_Males   # extracting the binomial random variables
sum(Male_Children$freq)     # summing all the frequencies
```

---

mazBETA *Beta Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Beta Distribution bounded between [0,1].

Usage

```
mazBETA(r,a,b)
```

Arguments

r	vector of moments.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.

Details

The probability density function and cumulative density function of a unit bounded beta distribution with random variable P are given by

$$g_P(p) = \frac{p^{a-1}(1-p)^{b-1}}{B(a,b)}$$

; $0 \leq p \leq 1$

$$G_P(p) = \frac{B_p(a,b)}{B(a,b)}$$

; $0 \leq p \leq 1$

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{a}{a+b}$$

$$var[P] = \frac{ab}{(a+b)^2(a+b+1)}$$

The moments about zero is denoted as

$$E[P^r] = \prod_{i=0}^{r-1}\left(\frac{a+i}{a+b+i}\right)$$

$r = 1, 2, 3, ...$

Defined as $B_p(a, b) = \int_0^p t^{a-1}(1-t)^{b-1}\,dt$ is incomplete beta integrals and $B(a, b)$ is the beta function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `mazBETA` gives the moments about zero in vector form.

References

Johnson NL, Kotz S, Balakrishnan N (1995). *Continuous univariate distributions, volume 2*, volume 289. John wiley and sons. Trenkler G (1996). "Continuous univariate distributions." *Computational Statistics and Data Analysis*, **21**(1), 119–119.

See Also

`Beta`

or

https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Beta.html

Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dBETA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dBETA(seq(0,1,by=0.01),2,3)$pdf   #extracting the pdf values
dBETA(seq(0,1,by=0.01),2,3)$mean  #extracting the mean
dBETA(seq(0,1,by=0.01),2,3)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pBETA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pBETA(seq(0,1,by=0.01),2,3)   #acquiring the cumulative probability values
mazBETA(1.4,3,2)              #acquiring the moment about zero values
```

```
mazBETA(2,3,2)-mazBETA(1,3,2)^2 #acquiring the variance for a=3,b=2

#only the integer value of moments is taken here because moments cannot be decimal
mazBETA(1.9,5.5,6)
```

`mazGAMMA` *Gamma Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for Gamma Distribution bounded between [0,1].

Usage

```
mazGAMMA(r,c,l)
```

Arguments

`r`	vector of moments.
`c`	single value for shape parameter c.
`l`	single value for shape parameter l.

Details

The probability density function and cumulative density function of a unit bounded Gamma distribution with random variable P are given by

$$g_P(p) = \frac{c^l p^{c-1}}{\gamma(l)}[ln(1/p)]^{l-1}$$

; $0 \leq p \leq 1$

$$G_P(p) = \frac{Ig(l, cln(1/p))}{\gamma(l)}$$

; $0 \leq p \leq 1$

$$l, c > 0$$

The mean the variance are denoted by

$$E[P] = (\frac{c}{c+1})^l$$

$$var[P] = (\frac{c}{c+2})^l - (\frac{c}{c+1})^{2l}$$

The moments about zero is denoted as

$$E[P^r] = (\frac{c}{c+r})^l$$

$r = 1, 2, 3, ...$

Defined as $\gamma(l)$ is the gamma function. Defined as $Ig(l, cln(1/p)) = \int_0^{cln(1/p)} t^{l-1} e^{-t} dt$ is the Lower incomplete gamma function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `mazGAMMA` gives the moments about zero in vector form.

References

Olshen AC (1938). "Transformations of the pearson type III distribution." *The Annals of Mathematical Statistics*, **9**(3), 176–200.

See Also

`GammaDist`

Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dGAMMA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dGAMMA(seq(0,1,by=0.01),5,6)$pdf   #extracting the pdf values
dGAMMA(seq(0,1,by=0.01),5,6)$mean  #extracting the mean
dGAMMA(seq(0,1,by=0.01),5,6)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pGAMMA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pGAMMA(seq(0,1,by=0.01),5,6)   #acquiring the cumulative probability values
mazGAMMA(1.4,5,6)              #acquiring the moment about zero values
mazGAMMA(2,5,6)-mazGAMMA(1,5,6)^2 #acquiring the variance for a=5,b=6

#only the integer value of moments is taken here because moments cannot be decimal
mazGAMMA(1.9,5.5,6)
```

`mazGBeta1` *Generalized Beta Type-1 Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Generalized Beta Type-1 Distribution bounded between [0,1].

Usage

```
mazGBeta1(r,a,b,c)
```

Arguments

`r`	vector of moments
`a`	single value for shape parameter alpha representing as a.
`b`	single value for shape parameter beta representing as b.
`c`	single value for shape parameter gamma representing as c.

Details

The probability density function and cumulative density function of a unit bounded Generalized Beta Type-1 Distribution with random variable P are given by

$$g_P(p) = \frac{c}{B(a,b)} p^{ac-1} (1-p^c)^{b-1}$$

; $0 \le p \le 1$

$$G_P(p) = \frac{p^{ac}}{aB(a,b)} 2F1(a, 1-b; p^c; a+1)$$

$0 \le p \le 1$

$$a, b, c > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$var[P] = \frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - (\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})})^2$$

The moments about zero is denoted as

$$E[P^r] = \frac{B(a+b, \frac{r}{c})}{B(a, \frac{r}{c})}$$

$r = 1, 2, 3, ...$

Defined as $B(a, b)$ is Beta function. Defined as $2F1(a, b; c; d)$ is Gaussian Hypergeometric function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output `mazGBeta1` gives the moments about zero in vector form.

References

Manoj C, Wijekoon P, Yapa RD (2013). "The McDonald generalized beta-binomial distribution: A new binomial mixture distribution and simulation based comparison with its nested distributions in handling overdispersion." *International journal of statistics and probability*, **2**(2), 24. Janiffer NM, Islam A, Luke O, others (2014). "Estimating Equations for Estimation of Mcdonald Generalized Beta—Binomial Parameters." *Open Journal of Statistics*, **4**(09), 702. Roozegar R, Tahmasebi S, Jafari AA (2017). "The McDonald Gompertz distribution: properties and applications." *Communications in Statistics-Simulation and Computation*, **46**(5), 3341–3355.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
lines(seq(0,1,by=0.001),dGBeta1(seq(0,1,by=0.001),a[i],1,2*a[i])$pdf,col = col[i])
}

dGBeta1(seq(0,1,by=0.01),2,3,1)$pdf    #extracting the pdf values
dGBeta1(seq(0,1,by=0.01),2,3,1)$mean   #extracting the mean
dGBeta1(seq(0,1,by=0.01),2,3,1)$var    #extracting the variance

pGBeta1(0.04,2,3,4)        #acquiring the cdf values for a=2,b=3,c=4
mazGBeta1(1.4,3,2,2)              #acquiring the moment about zero values
mazGBeta1(2,3,2,2)-mazGBeta1(1,3,2,2)^2        #acquiring the variance for a=3,b=2,c=2

#only the integer value of moments is taken here because moments cannot be decimal
mazGBeta1(3.2,3,2,2)
```

`mazGHGBeta` *Gaussian Hypergeometric Generalized Beta Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Gaussian Hypergeometric Generalized Beta distribution bounded between [0,1].

Usage

```
mazGHGBeta(r,n,a,b,c)
```

Arguments

r	vector of moments.
n	single value for no of binomial trials.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter lambda representing as c.

Details

The probability density function and cumulative density function of a unit bounded Gaussian Hypergeometric Generalized Beta Distribution with random variable P are given by

$$g_P(p) = \frac{1}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1}(1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}}$$

$; 0 \leq p \leq 1$

$$G_P(p) = \int_0^p \frac{1}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} t^{a-1}(1-t)^{b-1} \frac{c^{b+n}}{(c+(1-c)t)^{a+b+n}}\, dt$$

$; 0 \leq p \leq 1$

$$a, b, c > 0$$

$$n = 1, 2, 3, ...$$

The mean and the variance are denoted by

$$E[P] = \int_0^1 \frac{p}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1}(1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}}\, dp$$

$$var[P] = \int_0^1 \frac{p^2}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1}(1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}}\, dp - (E[p])^2$$

The moments about zero is denoted as

$$E[P^r] = \int_0^1 \frac{p^r}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1}(1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}}\, dp$$

$r = 1, 2, 3, ...$

Defined as $B(a,b)$ as the beta function. Defined as $2F1(a,b;c;d)$ as the Gaussian Hypergeometric function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of mazGHGBeta give the moments about zero in vector form.

References

Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ, Olmo-Jimenez MJ (2007). “A generalization of the beta–binomial distribution.” *Journal of the Royal Statistical Society Series C: Applied Statistics*, **56**(1), 51–61. Pearson JW (2009). *Computation of hypergeometric functions*. Ph.D. thesis, University of Oxford.

See Also

hypergeo_powerseries

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
lines(seq(0,1,by=0.001),dGHGBeta(seq(0,1,by=0.001),7,1+a[i],0.3,1+a[i])$pdf,col = col[i])
}

dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$pdf   #extracting the pdf values
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$mean  #extracting the mean
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(6)
a <- c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:6)
{
lines(seq(0.01,1,by=0.001),pGHGBeta(seq(0.01,1,by=0.001),7,1+a[i],0.3,1+a[i]),col=col[i])
}

pGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659) #acquiring the cumulative probability values
mazGHGBeta(1.4,7,1.6312,0.3913,0.6659)           #acquiring the moment about zero values

#acquiring the variance for a=1.6312,b=0.3913,c=0.6659
mazGHGBeta(2,7,1.6312,0.3913,0.6659)-mazGHGBeta(1,7,1.6312,0.3913,0.6659)^2

#only the integer value of moments is taken here because moments cannot be decimal
mazGHGBeta(1.9,15,5,6,1)
```

`mazKUM` *Kumaraswamy Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Kumaraswamy Distribution bounded between [0,1].

Usage

```
mazKUM(r,a,b)
```

Arguments

`r`	vector of moments.
`a`	single value for shape parameter alpha representing as a.
`b`	single value for shape parameter beta representing as b.

Details

The probability density function and cumulative density function of a unit bounded Kumaraswamy Distribution with random variable P are given by

$$g_P(p) = abp^{a-1}(1-p^a)^{b-1}$$

; $0 \leq p \leq 1$

$$G_P(p) = 1-(1-p^a)^b$$

; $0 \leq p \leq 1$

$$a,b > 0$$

The mean and the variance are denoted by

$$E[P] = bB(1+\frac{1}{a},b)$$

$$var[P] = bB(1+\frac{2}{a},b) - (bB(1+\frac{1}{a},b))^2$$

The moments about zero is denoted as

$$E[P^r] = bB(1+\frac{r}{a},b)$$

$r = 1,2,3,...$

Defined as $B(a,b)$ is the beta function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `mazKUM` gives the moments about zero in vector form.

References

Kumaraswamy P (1980). "A generalized probability density function for double-bounded random processes." *Journal of hydrology*, **46**(1-2), 79–88. Jones MC (2009). "Kumaraswamy's distribution: A beta-type distribution with some tractability advantages." *Statistical methodology*, **6**(1), 70–81.

Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,6))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dKUM(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dKUM(seq(0,1,by=0.01),2,3)$pdf   #extracting the probability values
dKUM(seq(0,1,by=0.01),2,3)$mean  #extracting the mean
dKUM(seq(0,1,by=0.01),2,3)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pKUM(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pKUM(seq(0,1,by=0.01),2,3)    #acquiring the cumulative probability values
mazKUM(1.4,3,2)               #acquiring the moment about zero values
mazKUM(2,2,3)-mazKUM(1,2,3)^2  #acquiring the variance for a=2,b=3

#only the integer value of moments is taken here because moments cannot be decimal
mazKUM(1.9,5.5,6)
```

`mazTRI` *Triangular Distribution Bounded Between [0,1]*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Triangular Distribution bounded between [0,1].

Usage

```
mazTRI(r,mode)
```

Arguments

r	vector of moments.
mode	single value for mode.

Details

Setting $min = 0$ and $max = 1$ $mode = c$ in the Triangular distribution a unit bounded Triangular distribution can be obtained. The probability density function and cumulative density function of a unit bounded Triangular distribution with random variable P are given by

$$g_P(p) = \frac{2p}{c}$$

; $0 \leq p < c$

$$g_P(p) = \frac{2(1-p)}{(1-c)}$$

; $c \leq p \leq 1$

$$G_P(p) = \frac{p^2}{c}$$

; $0 \leq p < c$

$$G_P(p) = 1 - \frac{(1-p)^2}{(1-c)}$$

; $c \leq p \leq 1$

$$0 \leq mode = c \leq 1$$

The mean and the variance are denoted by

$$E[P] = \frac{(a+b+c)}{3} = \frac{(1+c)}{3}$$

$$var[P] = \frac{a^2+b^2+c^2-ab-ac-bc}{18} = \frac{(1+c^2-c)}{18}$$

Moments about zero is denoted as

$$E[P^r] = \frac{2c^{r+2}}{c(r+2)} + \frac{2(1-c^{r+1})}{(1-c)(r+1)} + \frac{2(c^{r+2}-1)}{(1-c)(r+2)}$$

$r = 1, 2, 3, ...$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of mazTRI give the moments about zero in vector form.

References

Horsnell G (1957). “Economical acceptance sampling schemes.” *Journal of the Royal Statistical Society. Series A (General)*, **120**(2), 148–201. Johnson NL, Kotz S, Balakrishnan N (1995). *Continuous univariate distributions, volume 2*, volume 289. John wiley and sons. Karlis D, Xekalaki E (2008). *The polygonal distribution*. Springer. Okagbue HI, Edeki SO, Opanuga AA, Oguntunde PE, Adeosun ME (2014). “Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution.” *British Journal of Mathematics and Computer Science*, **4**(24), 3497.

Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
x <- seq(0.2,0.8,by=0.2)
plot(0,0,main="Probability density graph",xlab="Random variable",
ylab="Probability density values",xlim = c(0,1),ylim = c(0,3))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dTRI(seq(0,1,by=0.01),x[i])$pdf,col = col[i])
}

dTRI(seq(0,1,by=0.05),0.3)$pdf     #extracting the pdf values
dTRI(seq(0,1,by=0.01),0.3)$mean    #extracting the mean
dTRI(seq(0,1,by=0.01),0.3)$var     #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
x <- seq(0.2,0.8,by=0.2)
plot(0,0,main="Cumulative density graph",xlab="Random variable",
ylab="Cumulative density values",xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pTRI(seq(0,1,by=0.01),x[i]),col = col[i])
}

pTRI(seq(0,1,by=0.05),0.3)      #acquiring the cumulative probability values
mazTRI(1.4,.3)                  #acquiring the moment about zero values
mazTRI(2,.3)-mazTRI(1,.3)^2     #variance for when is mode 0.3

#only the integer value of moments is taken here because moments cannot be decimal
mazTRI(1.9,0.5)
```

mazUNI *Uniform Distribution Bounded Between [0,1]*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Uniform Distribution bounded between [0,1].

Usage

```
mazUNI(r)
```

Arguments

r	vector of moments

Details

Setting $a = 0$ and $b = 1$ in the Uniform Distribution a unit bounded Uniform Distribution can be obtained. The probability density function and cumulative density function of a unit bounded Uniform Distribution with random variable P are given by

$$g_P(p) = 1$$

$0 \leq p \leq 1$

$$G_P(p) = p$$

$0 \leq p \leq 1$

The mean and the variance are denoted as

$$E[P] = \frac{1}{a+b} = 0.5$$

$$var[P] = \frac{(b-a)^2}{12} = 0.0833$$

Moments about zero is denoted as

$$E[P^r] = \frac{e^{rb} - e^{ra}}{r(b-a)} = \frac{e^r - 1}{r}$$

$r = 1, 2, 3, ...$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `mazUNI` gives the moments about zero in vector form.

References

Horsnell G (1957). "Economical acceptance sampling schemes." *Journal of the Royal Statistical Society. Series A (General)*, **120**(2), 148–201. Johnson NL, Kotz S, Balakrishnan N (1995). *Continuous univariate distributions, volume 2*, volume 289. John wiley and sons.

See Also

Uniform

or

https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Uniform.html

Examples

```
#plotting the random variables and probability values
plot(seq(0,1,by=0.01),dUNI(seq(0,1,by=0.01))$pdf,type = "l",main="Probability density graph",
xlab="Random variable",ylab="Probability density values")

dUNI(seq(0,1,by=0.05))$pdf     #extract the pdf values
dUNI(seq(0,1,by=0.01))$mean    #extract the mean
dUNI(seq(0,1,by=0.01))$var     #extract the variance

#plotting the random variables and cumulative probability values
plot(seq(0,1,by=0.01),pUNI(seq(0,1,by=0.01)),type = "l",main="Cumulative density graph",
xlab="Random variable",ylab="Cumulative density values")

pUNI(seq(0,1,by=0.05))     #acquiring the cumulative probability values

mazUNI(c(1,2,3))    #acquiring the moment about zero values

#only the integer value of moments is taken here because moments cannot be decimal
mazUNI(1.9)
```

NegLLAddBin — *Negative Log Likelihood value of Additive Binomial distribution*

Description

This function will calculate the negative log likelihood value when the vector of binomial random variable and vector of corresponding frequencies are given with the input parameters.

Usage

```
NegLLAddBin(x,freq,p,alpha)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
p	single value for probability of success.
alpha	single value for alpha parameter.

Details

$$freq \geq 0$$
$$x = 0, 1, 2, ..$$
$$0 < p < 1$$
$$-1 < alpha < 1$$

Value

The output of NegLLAddBin will produce a single numeric value.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). "The use of a correlated binomial model for the analysis of certain toxicological experiments." *Biometrics*, 69–76. Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506. Morel JG, Neerchal NK (2012). *Overdispersion models in SAS*. SAS Publishing.

Examples

```
No.D.D <- 0:7          #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)         #assigning the corresponding frequencies

NegLLAddBin(No.D.D,Obs.fre.1,.5,.03)        #acquiring the negative log likelihood value
```

NegLLBetaBin — *Negative Log Likelihood value of Beta-Binomial Distribution*

Description

This function will calculate the Negative Log Likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters a and b.

Usage

```
NegLLBetaBin(x,freq,a,b)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.

Details

$$0 < a, b$$

$$freq \geq 0$$

$$x = 0, 1, 2, ...$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `NegLLBetaBin` will produce a single numeric value.

References

Young-Xu Y, Chan KA (2008). "Pooling overdispersed binomial data to estimate event rate." *BMC medical research methodology*, **8**, 1–12. Trenkler G (1996). "Continuous univariate distributions." *Computational Statistics and Data Analysis*, **21**(1), 119–119. HUGHES G, MADDEN L (1993). "Using the beta-binomial distribution to describe aggegated patterns of disease incidence." *Phytopathology*, **83**(7), 759–763.

Examples

```
No.D.D <- 0:7       #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

NegLLBetaBin(No.D.D,Obs.fre.1,.3,.4)   #acquiring the negative log likelihood value
```

NegLLBetaCorrBin *Negative Log Likelihood value of Beta-Correlated Binomial distribution*

Description

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the input parameters.

Usage

```
NegLLBetaCorrBin(x,freq,cov,a,b)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
cov	single value for covariance.
a	single value for alpha parameter.
b	single value for beta parameter.

Details

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$-\infty < cov < +\infty$$

$$0 < a, b$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `NegLLBetaCorrBin` will produce a single numeric value.

References

Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506.

Examples

```
No.D.D <- 0:7         #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies

NegLLBetaCorrBin(No.D.D,Obs.fre.1,0.001,9.03,10)    #acquiring the negative log likelihood value
```

NegLLCOMPBin *Negative Log Likelihood value of COM Poisson Binomial distribution*

Description

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the input parameters.

Usage

```
NegLLCOMPBin(x,freq,p,v)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
p	single value for probability of success.
v	single value for v.

Details

$$freq \geq 0$$
$$x = 0, 1, 2, ..$$
$$0 < p < 1$$
$$-\infty < v < +\infty$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `NegLLCOMPBin` will produce a single numeric value.

References

Borges P, Rodrigues J, Balakrishnan N, Bazan J (2014). "A COM–Poisson type generalization of the binomial distribution and its properties and applications." *Statistics and Probability Letters*, **87**, 158–166.

Examples

```
No.D.D <- 0:7         #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies

NegLLCOMPBin(No.D.D,Obs.fre.1,.5,.03)     #acquiring the negative log likelihood value
```

NegLLCorrBin *Negative Log Likelihood value of Correlated Binomial distribution*

Description

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the input parameters.

Usage

```
NegLLCorrBin(x,freq,p,cov)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
p	single value for probability of success.
cov	single value for covariance.

Details

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$-\infty < cov < +\infty$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `NegLLCorrBin` will produce a single numeric value.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). "The use of a correlated binomial model for the analysis of certain toxicological experiments." *Biometrics*, 69–76. Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506. Morel JG, Neerchal NK (2012). *Overdispersion models in SAS*. SAS Publishing.

Examples

```
No.D.D <- 0:7         #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)      #assigning the corresponding frequencies

NegLLCorrBin(No.D.D,Obs.fre.1,.5,.03)     #acquiring the negative log likelihood value
```

`NegLLGammaBin` *Negative Log Likelihood value of Gamma Binomial Distribution*

Description

This function will calculate the Negative Log Likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters l and c.

Usage

```
NegLLGammaBin(x,freq,c,l)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
c	single value for shape parameter c.
l	single value for shape parameter l.

Details

$$0 < l, c$$

$$freq \geq 0$$

$$x = 0, 1, 2, ...$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `NegLLGammaBin` will produce a single numeric value.

References

Grassia A (1977). "On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions." *Australian Journal of Statistics*, **19**(2), 108–114.

Examples

```
No.D.D <- 0:7       #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

NegLLGammaBin(No.D.D,Obs.fre.1,.3,.4)   #acquiring the negative log likelihood value
```

NegLLGHGBB — *Negative Log Likelihood value of Gaussian Hypergeometric Generalized Beta Binomial Distribution*

Description

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters a,b and c.

Usage

```
NegLLGHGBB(x,freq,a,b,c)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter alpha representing a.
b	single value for shape parameter beta representing b.
c	single value for shape parameter lambda representing c.

Details

$$0 < a, b, c$$

$$freq \geq 0$$

$$x = 0, 1, 2, ...$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of NegLLGHGBB will produce a single numeric value.

References

Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ, Olmo-Jimenez MJ (2007). "A generalization of the beta–binomial distribution." *Journal of the Royal Statistical Society Series C: Applied Statistics*, **56**(1), 51–61. Pearson JW (2009). *Computation of hypergeometric functions*. Ph.D. thesis, University of Oxford.

See Also

hypergeo_powerseries

Examples

```
No.D.D <- 0:7                    #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)     #assigning the corresponding frequencies
NegLLGHGBB(No.D.D,Obs.fre.1,.2,.3,1)     #acquiring the negative log likelihood value
```

NegLLGrassiaIIBin *Negative Log Likelihood value of Grassia II Binomial Distribution*

Description

This function will calculate the Negative Log Likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters l and c.

Usage

```
NegLLGrassiaIIBin(x,freq,a,b)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter a.
b	single value for shape parameter b.

Details

$$0 < a, b$$

$$freq \geq 0$$

$$x = 0, 1, 2, ...$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of NegLLGrassiaIIBin will produce a single numeric value.

References

Grassia A (1977). "On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions." *Australian Journal of Statistics*, **19**(2), 108–114.

Examples

```
No.D.D <- 0:7       #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)   #assigning the corresponding frequencies

NegLLGrassiaIIBin(No.D.D,Obs.fre.1,.3,.4)   #acquiring the negative log likelihood value
```

NegLLKumBin — *Negative Log Likelihood value of Kumaraswamy Binomial Distribution*

Description

This function will calculate the Negative Log Likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters a and b and iterations it.

Usage

```
NegLLKumBin(x,freq,a,b,it=25000)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
it	number of iterations to converge as a proper probability function replacing infinity.

Details

$$0 < a, b$$

$$x = 0, 1, 2, ...$$

$$freq \geq 0$$

$$it > 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `NegLLKumBin` will produce a single numeric value.

References

Xiaohu L, Yanyan H, Xueyan Z (2011). "The Kumaraswamy binomial distribution." *Chinese Journal of Applied Probability and Statistics*, **27**(5), 511–521.

Examples

```
No.D.D <- 0:7          #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

## Not run:
NegLLKumBin(No.D.D,Obs.fre.1,1.3,4.4) #acquiring the negative log likelihood value

## End(Not run)
```

`NegLLLMBin` *Negative Log Likelihood value of Lovinson Multiplicative Binomial distribution*

Description

This function will calculate the negative log likelihood value when the vector of binomial random variable and vector of corresponding frequencies are given with the input parameters.

Usage

```
NegLLLMBin(x,freq,p,phi)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
p	single value for probability of success.
phi	single value for phi parameter.

Details

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$0 < phi$$

Value

The output of `NegLLLMBin` will produce a single numeric value.

References

Elamir EA (2013). "Multiplicative-Binomial Distribution: Some Results on Characterization, Inference and Random Data Generation." *Journal of Statistical Theory and Applications*, **12**(1), 92–105.

Examples

```
No.D.D <- 0:7       #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies

NegLLLMBin(No.D.D,Obs.fre.1,.5,3)    #acquiring the negative log likelihood value
```

NegLLMcGBB — *Negative Log Likelihood value of McDonald Generalized Beta Binomial Distribution*

Description

This function will calculate the negative log likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the shape parameters a,b and c.

Usage

```
NegLLMcGBB(x,freq,a,b,c)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter gamma representing as c.

Details

$$0 < a, b, c$$

$$freq \geq 0$$

$$x = 0, 1, 2, ...$$

Value

The output of NegLLMcGBB will produce a single numeric value.

References

Manoj C, Wijekoon P, Yapa RD (2013). "The McDonald generalized beta-binomial distribution: A new binomial mixture distribution and simulation based comparison with its nested distributions in handling overdispersion." *International journal of statistics and probability*, **2**(2), 24. Janiffer NM, Islam A, Luke O, others (2014). "Estimating Equations for Estimation of Mcdonald Generalized Beta—Binomial Parameters." *Open Journal of Statistics*, **4**(09), 702. Roozegar R, Tahmasebi S, Jafari AA (2017). "The McDonald Gompertz distribution: properties and applications." *Communications in Statistics-Simulation and Computation*, **46**(5), 3341–3355.

Examples

```
No.D.D <- 0:7            #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies

NegLLMcGBB(No.D.D,Obs.fre.1,.2,.3,1)    #acquiring the negative log likelihood value
```

NegLLMultiBin   *Negative Log Likelihood value of Multiplicative Binomial distribution*

Description

This function will calculate the negative log likelihood value when the vector of binomial random variable and vector of corresponding frequencies are given with the input parameters.

Usage

```
NegLLMultiBin(x,freq,p,theta)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
p	single value for probability of success.
theta	single value for theta parameter.

Details

$$freq \geq 0$$

$$x = 0, 1, 2, ..$$

$$0 < p < 1$$

$$0 < theta$$

Value

The output of NegLLMultiBin will produce a single numeric value.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). "The use of a correlated binomial model for the analysis of certain toxicological experiments." *Biometrics*, 69–76. Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506.

Examples

```
No.D.D <- 0:7       #assigning the random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95)    #assigning the corresponding frequencies

NegLLMultiBin(No.D.D,Obs.fre.1,.5,3)    #acquiring the negative log likelihood value
```

NegLLTriBin *Negative Log Likelihood value of Triangular Binomial Distribution*

Description

This function will calculate the Negative Log Likelihood value when the vector of binomial random variables and vector of corresponding frequencies are given with the mode value.

Usage

```
NegLLTriBin(x,freq,mode)
```

Arguments

x	vector of binomial random variables.
freq	vector of frequencies.
mode	single value for mode.

Details

$$0 < mode = c < 1$$
$$x = 0, 1, 2, , ...$$
$$freq \geq 0$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `NegLLTriBin` will produce a single numeric value.

References

Horsnell G (1957). "Economical acceptance sampling schemes." *Journal of the Royal Statistical Society. Series A (General)*, **120**(2), 148–201. Karlis D, Xekalaki E (2008). *The polygonal distribution*. Springer. Okagbue HI, Edeki SO, Opanuga AA, Oguntunde PE, Adeosun ME (2014). "Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution." *British Journal of Mathematics and Computer Science*, **4**(24), 3497.

Examples

```
No.D.D <- 0:7    #assigning the Random variables
Obs.fre.1 <- c(47,54,43,40,40,41,39,95) #assigning the corresponding frequencies

NegLLTriBin(No.D.D,Obs.fre.1,.023)   #acquiring the Negative log likelihood value
```

Overdispersion *Overdispersion*

Description

After fitting the distribution using this function we can extract the overdispersion value. This function works for fitTriBin, fitBetaBin, fitKumBin, fitGHGBB and fitMcGBB for Binomial Mixture Distributions. Similarly, Alternate Binomial Distributions also support this function for fitAddBin,fitBetaCorrBin, fitCOMPBin, fitCorrBin and fitMultiBin.

Usage

```
Overdispersion(object)
```

Arguments

object	An object from one of the classes of fitTB,fitBB,fitKB,fitGB,fitMB.

Details

Note : Only objects from classes of above mentioned classes can be used.

Value

The output of `Overdispersion` gives a single value which is the overdispersion.

Examples

```
No.D.D=0:7      #assigning the random variables
Obs.fre.1=c(47,54,43,40,40,41,39,95)  #assigning the corresponding frequencies

#estimating mode value for given data
results<-EstMLETriBin(No.D.D,Obs.fre.1)
results
mode<-results$mode

#fitting the Triangular Bionomial distribution for estimated parameters
TriBin<-fitTriBin(No.D.D,Obs.fre.1,mode)
TriBin

#extracting the overdispersion
Overdispersion(TriBin)
```

pAddBin *Additive Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Additive Binomial Distribution.

Usage

```
pAddBin(x,n,p,alpha)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
p	single value for probability of success.
alpha	single value for alpha parameter.

Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{AddBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} (\frac{alpha}{2}(\frac{x(x-1)}{p} + \frac{(n-x)(n-x-1)}{(1-p)} - \frac{alphan(n-1)}{2}) + 1)$$

$$x = 0, 1, 2, 3, ...n$$

$$n = 1, 2, 3, ...$$

$$0 < p < 1$$

$$-1 < alpha < 1$$

The alpha is in between

$$\frac{-2}{n(n-1)} min(\frac{p}{1-p}, \frac{1-p}{p}) \leq alpha \leq (\frac{n + (2p-1)^2}{4p(1-p)})^{-1}$$

The mean and the variance are denoted as

$$E_{Addbin}[x] = np$$

$$Var_{Addbin}[x] = np(1-p)(1 + (n-1)alpha)$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of pAddBin gives cumulative probability values in vector form.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). "The use of a correlated binomial model for the analysis of certain toxicological experiments." *Biometrics*, 69–76. Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506. Morel JG, Neerchal NK (2012). *Overdispersion models in SAS*. SAS Publishing.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Additive binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
  lines(0:10,dAddBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
  points(0:10,dAddBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}

dAddBin(0:10,10,0.58,0.022)$pdf     #extracting the probability values
dAddBin(0:10,10,0.58,0.022)$mean    #extracting the mean
dAddBin(0:10,10,0.58,0.022)$var     #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Additive binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pAddBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pAddBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}

pAddBin(0:10,10,0.58,0.022)       #acquiring the cumulative probability values
```

---

pBETA *Beta Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Beta Distribution bounded between [0,1].

Usage

```
pBETA(p,a,b)
```

Arguments

p	vector of probabilities.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.

Details

The probability density function and cumulative density function of a unit bounded beta distribution with random variable P are given by

$$g_P(p) = \frac{p^{a-1}(1-p)^{b-1}}{B(a,b)}$$

; $0 \leq p \leq 1$

$$G_P(p) = \frac{B_p(a,b)}{B(a,b)}$$

; $0 \leq p \leq 1$

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{a}{a+b}$$

$$var[P] = \frac{ab}{(a+b)^2(a+b+1)}$$

The moments about zero is denoted as

$$E[P^r] = \prod_{i=0}^{r-1}\left(\frac{a+i}{a+b+i}\right)$$

$r = 1, 2, 3, ...$

Defined as $B_p(a,b) = \int_0^p t^{a-1}(1-t)^{b-1}\,dt$ is incomplete beta integrals and $B(a,b)$ is the beta function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of pBETA gives the cumulative density values in vector form.

References

Johnson NL, Kotz S, Balakrishnan N (1995). *Continuous univariate distributions, volume 2*, volume 289. John wiley and sons. Trenkler G (1996). "Continuous univariate distributions." *Computational Statistics and Data Analysis*, **21**(1), 119–119.

See Also

Beta

or

https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Beta.html

Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dBETA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dBETA(seq(0,1,by=0.01),2,3)$pdf   #extracting the pdf values
dBETA(seq(0,1,by=0.01),2,3)$mean  #extracting the mean
dBETA(seq(0,1,by=0.01),2,3)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pBETA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pBETA(seq(0,1,by=0.01),2,3)   #acquiring the cumulative probability values
mazBETA(1.4,3,2)              #acquiring the moment about zero values
mazBETA(2,3,2)-mazBETA(1,3,2)^2 #acquiring the variance for a=3,b=2

#only the integer value of moments is taken here because moments cannot be decimal
mazBETA(1.9,5.5,6)
```

---

pBetaBin *Beta-Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Beta-Binomial Distribution.

Usage

```
pBetaBin(x,n,a,b)
```

Arguments

`x`	vector of binomial random variables.
`n`	single value for no of binomial trials.
`a`	single value for shape parameter alpha representing as a.
`b`	single value for shape parameter beta representing as b.

Details

Mixing Beta distribution with Binomial distribution will create the Beta-Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{BetaBin}(x) = \binom{n}{x} \frac{B(a+x, n+b-x)}{B(a,b)}$$

$$a, b > 0$$

$$x = 0, 1, 2, 3, ...n$$

$$n = 1, 2, 3, ...$$

The mean, variance and over dispersion are denoted as

$$E_{BetaBin}[x] = \frac{na}{a+b}$$

$$Var_{BetaBin}[x] = \frac{(nab)}{(a+b)^2} \frac{(a+b+n)}{(a+b+1)}$$

$$overdispersion = \frac{1}{a+b+1}$$

Defined as `B(a,b)` is the beta function.

Value

The output of `pBetaBin` gives cumulative probability values in vector form.

References

Young-Xu Y, Chan KA (2008). "Pooling overdispersed binomial data to estimate event rate." *BMC medical research methodology*, **8**, 1–12. Trenkler G (1996). "Continuous univariate distributions." *Computational Statistics and Data Analysis*, **21**(1), 119–119. HUGHES G, MADDEN L (1993). "Using the beta-binomial distribution to describe aggegated patterns of disease incidence." *Phytopathology*, **83**(7), 759–763.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.2)
plot(0,0,main="Beta-binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dBetaBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dBetaBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

dBetaBin(0:10,10,4,.2)$pdf    #extracting the pdf values
dBetaBin(0:10,10,4,.2)$mean   #extracting the mean
dBetaBin(0:10,10,4,.2)$var    #extracting the variance
dBetaBin(0:10,10,4,.2)$over.dis.para  #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pBetaBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pBetaBin(0:10,10,a[i],a[i]),col = col[i])
}

pBetaBin(0:10,10,4,.2)   #acquiring the cumulative probability values
```

`pBetaCorrBin` *Beta-Correlated Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Beta-Correlated Binomial Distribution.

Usage

```
pBetaCorrBin(x,n,cov,a,b)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
cov	single value for covariance.
a	single value for alpha parameter
b	single value for beta parameter.

Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{BetaCorrBin}(x) = \binom{n}{x}\frac{B(a+x,b+n-x)}{B(a+b)}\Bigg[1+\frac{cov}{2}\Bigg(\frac{\left(x(x-1)\prod_{k=1}^{4}(a+b+n-k)\right)}{\left(\prod_{k=1}^{2}(x+a-k)\prod_{k=1}^{2}(n-x+b-k)\right)}$$
$$-\frac{\left(2x(n-1)\prod_{k=1}^{3}(a+b+n-k)\right)}{\left((x+a-1)\prod_{k=1}^{2}(n-x+b-k)\right)}+\frac{\left(n(n-1)\prod_{k=1}^{2}(a+b+n-k)\right)}{\left(\prod_{k=1}^{2}(n-x+b-k)\right)}\Bigg)\Bigg]$$

$$x = 0, 1, 2, 3, ...n$$
$$n = 1, 2, 3, ...$$
$$-\infty < cov < +\infty$$
$$0 < a, b$$
$$0 < p < 1$$

$$p = \frac{a}{a+b}$$
$$\Theta = \frac{1}{a+b}$$

The Correlation is in between

$$\frac{-2}{n(n-1)}min(\frac{p}{1-p}, \frac{1-p}{p}) \leq correlation \leq \frac{2p(1-p)}{(n-1)p(1-p)+0.25-fo}$$

where $fo = min(x-(n-1)p-0.5)^2$

The mean and the variance are denoted as

$$E_{BetaCorrBin}[x] = np$$
$$Var_{BetaCorrBin}[x] = np(1-p)(n\Theta+1)(1+\Theta)^{-1} + n(n-1)cov$$
$$Corr_{BetaCorrBin}[x] = \frac{cov}{p(1-p)}$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of pBetaCorrBin gives cumulative probability values in vector form.

References

Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(9.0,10,11,12,13)
b <- c(8.0,8.1,8.2,8.3,8.4)
plot(0,0,main="Beta-Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dBetaCorrBin(0:10,10,0.001,a[i],b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dBetaCorrBin(0:10,10,0.001,a[i],b[i])$pdf,col = col[i],pch=16)
}

dBetaCorrBin(0:10,10,0.001,10,13)$pdf      #extracting the pdf values
dBetaCorrBin(0:10,10,0.001,10,13)$mean     #extracting the mean
dBetaCorrBin(0:10,10,0.001,10,13)$var      #extracting the variance
dBetaCorrBin(0:10,10,0.001,10,13)$corr     #extracting the correlation
dBetaCorrBin(0:10,10,0.001,10,13)$mincorr  #extracting the minimum correlation value
dBetaCorrBin(0:10,10,0.001,10,13)$maxcorr  #extracting the maximum correlation value

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(9.0,10,11,12,13)
b <- c(8.0,8.1,8.2,8.3,8.4)
plot(0,0,main="Beta-Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pBetaCorrBin(0:10,10,0.001,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pBetaCorrBin(0:10,10,0.001,a[i],b[i]),col = col[i],pch=16)
}

pBetaCorrBin(0:10,10,0.001,10,13)      #acquiring the cumulative probability values
```

---

pCOMPBin *COM Poisson Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the COM Poisson Binomial Distribution.

Usage

```
pCOMPBin(x,n,p,v)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
p	single value for probability of success.
v	single value for v.

Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{COMPBin}(x) = \frac{{n \choose x}^v p^x (1-p)^{n-x}}{\sum_{j=0}^{n} {n \choose j}^v p^j (1-p)^{(n-j)}}$$

$$x = 0, 1, 2, 3, ...n$$

$$n = 1, 2, 3, ...$$

$$0 < p < 1$$

$$-\infty < v < +\infty$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `pCOMPBin` gives cumulative probability values in vector form.

References

Borges P, Rodrigues J, Balakrishnan N, Bazan J (2014). “A COM–Poisson type generalization of the binomial distribution and its properties and applications.” *Statistics and Probability Letters*, **87**, 158–166.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="COM Poisson Binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dCOMPBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
```

```
points(0:10,dCOMPBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}

dCOMPBin(0:10,10,0.58,0.022)$pdf      #extracting the pdf values
dCOMPBin(0:10,10,0.58,0.022)$mean     #extracting the mean
dCOMPBin(0:10,10,0.58,0.022)$var      #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="COM Poisson Binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pCOMPBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pCOMPBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}

pCOMPBin(0:10,10,0.58,0.022)      #acquiring the cumulative probability values
```

---

pCorrBin *Correlated Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Correlated Binomial Distribution.

Usage

```
pCorrBin(x,n,p,cov)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
p	single value for probability of success.
cov	single value for covariance.

Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{CorrBin}(x) = \binom{n}{x}(p^x)(1-p)^{n-x}(1+(\frac{cov}{2p^2(1-p)^2})((x-np)^2+x(2p-1)-np^2))$$

$x = 0, 1, 2, 3, ...n$ $n = 1, 2, 3, ...$ $0 < p < 1$ $-\infty < cov < +\infty$

The Correlation is in between

$$\frac{-2}{n(n-1)} min(\frac{p}{1-p}, \frac{1-p}{p}) \leq cov \leq \frac{2p(1-p)}{(n-1)p(1-p) + 0.25 - fo}$$

where $fo = min(x - (n-1)p - 0.5)^2$

The mean and the variance are denoted as

$$E_{CorrBin}[x] = np$$

$$Var_{CorrBin}[x] = n(p(1-p) + (n-1)cov)$$

$$Corr_{CorrBin}[x] = \frac{cov}{p(1-p)}$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `pCorrBin` gives cumulative probability values in vector form.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). "The use of a correlated binomial model for the analysis of certain toxicological experiments." *Biometrics*, 69–76. Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506. Morel JG, Neerchal NK (2012). *Overdispersion models in SAS*. SAS Publishing.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dCorrBin(0:10,10,a[i],b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dCorrBin(0:10,10,a[i],b[i])$pdf,col = col[i],pch=16)
}

dCorrBin(0:10,10,0.58,0.022)$pdf      #extracting the pdf values
dCorrBin(0:10,10,0.58,0.022)$mean     #extracting the mean
dCorrBin(0:10,10,0.58,0.022)$var      #extracting the variance
dCorrBin(0:10,10,0.58,0.022)$corr     #extracting the correlation
dCorrBin(0:10,10,0.58,0.022)$mincorr  #extracting the minimum correlation value
dCorrBin(0:10,10,0.58,0.022)$maxcorr  #extracting the maximum correlation value
```

```
#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Correlated binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pCorrBin(0:10,10,a[i],b[i]),col = col[i],lwd=2.85)
points(0:10,pCorrBin(0:10,10,a[i],b[i]),col = col[i],pch=16)
}

pCorrBin(0:10,10,0.58,0.022)      #acquiring the cumulative probability values
```

---

pGAMMA *Gamma Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for Gamma Distribution bounded between [0,1].

Usage

```
pGAMMA(p,c,l)
```

Arguments

p	vector of probabilities.
c	single value for shape parameter c.
l	single value for shape parameter l.

Details

The probability density function and cumulative density function of a unit bounded Gamma distribution with random variable P are given by

$$g_P(p) = \frac{c^l p^{c-1}}{\gamma(l)} [ln(1/p)]^{l-1}$$

; $0 \le p \le 1$

$$G_P(p) = \frac{Ig(l, cln(1/p))}{\gamma(l)}$$

; $0 \le p \le 1$

$$l, c > 0$$

The mean the variance are denoted by

$$E[P] = (\frac{c}{c+1})^l$$

$$var[P] = (\frac{c}{c+2})^l - (\frac{c}{c+1})^{2l}$$

The moments about zero is denoted as

$$E[P^r] = (\frac{c}{c+r})^l$$

$r = 1, 2, 3, ...$

Defined as $\gamma(l)$ is the gamma function. Defined as $Ig(l, cln(1/p)) = \int_0^{cln(1/p)} t^{l-1} e^{-t} dt$ is the Lower incomplete gamma function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `pGAMMA` gives the cumulative density values in vector form.

References

Olshen AC (1938). "Transformations of the pearson type III distribution." *The Annals of Mathematical Statistics*, **9**(3), 176–200.

See Also

GammaDist

Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,4))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dGAMMA(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}

dGAMMA(seq(0,1,by=0.01),5,6)$pdf   #extracting the pdf values
dGAMMA(seq(0,1,by=0.01),5,6)$mean  #extracting the mean
dGAMMA(seq(0,1,by=0.01),5,6)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
```

```
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pGAMMA(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pGAMMA(seq(0,1,by=0.01),5,6)   #acquiring the cumulative probability values
mazGAMMA(1.4,5,6)              #acquiring the moment about zero values
mazGAMMA(2,5,6)-mazGAMMA(1,5,6)^2 #acquiring the variance for a=5,b=6

#only the integer value of moments is taken here because moments cannot be decimal
mazGAMMA(1.9,5.5,6)
```

---

pGammaBin *Gamma Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Gamma Binomial Distribution.

Usage

```
pGammaBin(x,n,c,l)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
c	single value for shape parameter c.
l	single value for shape parameter l.

Details

Mixing Gamma distribution with Binomial distribution will create the the Gamma Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{GammaBin}[x] = \binom{n}{x} \sum_{j=0}^{n-x} \binom{n-x}{j} (-1)^j (\frac{c}{c+x+j})^l$$

$$c, l > 0$$

$$x = 0, 1, 2, ..., n$$

$$n = 1, 2, 3, ...$$

The mean, variance and over dispersion are denoted as

$$E_{GammaBin}[x] = (\frac{c}{c+1})^l$$

$$Var_{GammaBin}[x] = n^2[(\frac{c}{c+2})^l - (\frac{c}{c+1})^{2l}] + n(\frac{c}{c+1})^l 1-)(\frac{c+1}{c+2})^l$$

$$overdispersion = \frac{(\frac{c}{c+2})^l - (\frac{c}{c+1})^{2l}}{(\frac{c}{c+1})^l[1 - (\frac{c}{c+1})^l]}$$

Value

The output of `pGammaBin` gives cumulative probability values in vector form.

References

Grassia A (1977). “On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions.” *Australian Journal of Statistics*, **19**(2), 108–114.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.2)
plot(0,0,main="Gamma-binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dGammaBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dGammaBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
}

dGammaBin(0:10,10,4,.2)$pdf    #extracting the pdf values
dGammaBin(0:10,10,4,.2)$mean   #extracting the mean
dGammaBin(0:10,10,4,.2)$var    #extracting the variance
dGammaBin(0:10,10,4,.2)$over.dis.para  #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pGammaBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pGammaBin(0:10,10,a[i],a[i]),col = col[i])
}

pGammaBin(0:10,10,4,.2)   #acquiring the cumulative probability values
```

pGBeta1 *Generalized Beta Type-1 Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Generalized Beta Type-1 Distribution bounded between [0,1].

Usage

```
pGBeta1(p,a,b,c)
```

Arguments

p	vector of probabilities.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter gamma representing as c.

Details

The probability density function and cumulative density function of a unit bounded Generalized Beta Type-1 Distribution with random variable P are given by

$$g_P(p) = \frac{c}{B(a,b)} p^{ac-1} (1-p^c)^{b-1}$$

$; 0 \leq p \leq 1$

$$G_P(p) = \frac{p^{ac}}{aB(a,b)} 2F1(a, 1-b; p^c; a+1)$$

$0 \leq p \leq 1$

$$a, b, c > 0$$

The mean and the variance are denoted by

$$E[P] = \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$var[P] = \frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - (\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})})^2$$

The moments about zero is denoted as

$$E[P^r] = \frac{B(a+b, \frac{r}{c})}{B(a, \frac{r}{c})}$$

$r = 1, 2, 3, ...$

Defined as $B(a, b)$ is Beta function. Defined as $2F1(a, b; c; d)$ is Gaussian Hypergeometric function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output `pGBeta1` gives the cumulative density values in vector form.

References

Manoj C, Wijekoon P, Yapa RD (2013). "The McDonald generalized beta-binomial distribution: A new binomial mixture distribution and simulation based comparison with its nested distributions in handling overdispersion." *International journal of statistics and probability*, **2**(2), 24. Janiffer NM, Islam A, Luke O, others (2014). "Estimating Equations for Estimation of Mcdonald Generalized Beta—Binomial Parameters." *Open Journal of Statistics*, **4**(09), 702. Roozegar R, Tahmasebi S, Jafari AA (2017). "The McDonald Gompertz distribution: properties and applications." *Communications in Statistics-Simulation and Computation*, **46**(5), 3341–3355.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
lines(seq(0,1,by=0.001),dGBeta1(seq(0,1,by=0.001),a[i],1,2*a[i])$pdf,col = col[i])
}

dGBeta1(seq(0,1,by=0.01),2,3,1)$pdf    #extracting the pdf values
dGBeta1(seq(0,1,by=0.01),2,3,1)$mean   #extracting the mean
dGBeta1(seq(0,1,by=0.01),2,3,1)$var    #extracting the variance

pGBeta1(0.04,2,3,4)        #acquiring the cdf values for a=2,b=3,c=4
mazGBeta1(1.4,3,2,2)              #acquiring the moment about zero values
mazGBeta1(2,3,2,2)-mazGBeta1(1,3,2,2)^2        #acquiring the variance for a=3,b=2,c=2

#only the integer value of moments is taken here because moments cannot be decimal
mazGBeta1(3.2,3,2,2)
```

---

pGHGBB *Gaussian Hypergeometric Generalized Beta Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Gaussian Hypergeometric Generalized Beta Binomial distribution.

Usage

```
pGHGBB(x,n,a,b,c)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
a	single value for shape parameter alpha value representing a.
b	single value for shape parameter beta value representing b.
c	single value for shape parameter lambda value representing c.

Details

Mixing Gaussian Hypergeometric Generalized Beta distribution with Binomial distribution will create the Gaussian Hypergeometric Generalized Beta Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{GHGBB}(x) = \frac{1}{2F1(-n, a; -b-n+1; c)} \binom{n}{x} \frac{B(x+a, n-x+b)}{B(a, b+n)} (c^x)$$

$$a, b, c > 0$$

$$x = 0, 1, 2, ...n$$

$$n = 1, 2, 3, ...$$

The mean, variance and over dispersion are denoted as

$$E_{GHGBB}[x] = nE_{GHGBeta}$$

$$Var_{GHGBB}[x] = nE_{GHGBeta}(1 - E_{GHGBeta}) + n(n-1)Var_{GHGBeta}$$

$$overdispersion = \frac{var_{GHGBeta}}{E_{GHGBeta}(1 - E_{GHGBeta})}$$

Defined as $B(a, b)$ is the beta function. Defined as $2F1(a, b; c; d)$ is the Gaussian Hypergeometric function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `pGHGBB` gives cumulative probability function values in vector form.

References

Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ, Olmo-Jimenez MJ (2007). “A generalization of the beta–binomial distribution.” *Journal of the Royal Statistical Society Series C: Applied Statistics*, **56**(1), 51–61. Pearson JW (2009). *Computation of hypergeometric functions*. Ph.D. thesis, University of Oxford.

See Also

hypergeo_powerseries

Examples

```
#plotting the random variables and probability values
col <- rainbow(6)
a <- c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="GHGBB probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,7),ylim = c(0,0.9))
for (i in 1:6)
{
lines(0:7,dGHGBB(0:7,7,1+a[i],0.3,1+a[i])$pdf,col = col[i],lwd=2.85)
points(0:7,dGHGBB(0:7,7,1+a[i],0.3,1+a[i])$pdf,col = col[i],pch=16)
}

dGHGBB(0:7,7,1.3,0.3,1.3)$pdf      #extracting the pdf values
dGHGBB(0:7,7,1.3,0.3,1.3)$mean     #extracting the mean
dGHGBB(0:7,7,1.3,0.3,1.3)$var      #extracting the variance
dGHGBB(0:7,7,1.3,0.3,1.3)$over.dis.par  #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,7),ylim = c(0,1))
for (i in 1:4)
{
lines(0:7,pGHGBB(0:7,7,1+a[i],0.3,1+a[i]),col = col[i])
points(0:7,pGHGBB(0:7,7,1+a[i],0.3,1+a[i]),col = col[i])
}

pGHGBB(0:7,7,1.3,0.3,1.3)     #acquiring the cumulative probability values
```

---

pGHGBeta *Gaussian Hypergeometric Generalized Beta Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Gaussian Hypergeometric Generalized Beta distribution bounded between [0,1].

Usage

```
pGHGBeta(p,n,a,b,c)
```

Arguments

p	vector of probabilities.
n	single value for no of binomial trials.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter lambda representing as c.

Details

The probability density function and cumulative density function of a unit bounded Gaussian Hypergeometric Generalized Beta Distribution with random variable P are given by

$$g_P(p) = \frac{1}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1}(1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}}$$

$; 0 \le p \le 1$

$$G_P(p) = \int_0^p \frac{1}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} t^{a-1}(1-t)^{b-1} \frac{c^{b+n}}{(c+(1-c)t)^{a+b+n}} \, dt$$

$; 0 \le p \le 1$

$$a, b, c > 0$$

$$n = 1, 2, 3, ...$$

The mean and the variance are denoted by

$$E[P] = \int_0^1 \frac{p}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1}(1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}} \, dp$$

$$var[P] = \int_0^1 \frac{p^2}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1}(1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}} \, dp - (E[p])^2$$

The moments about zero is denoted as

$$E[P^r] = \int_0^1 \frac{p^r}{B(a,b)} \frac{2F1(-n,a;-b-n+1;1)}{2F1(-n,a;-b-n+1;c)} p^{a-1}(1-p)^{b-1} \frac{c^{b+n}}{(c+(1-c)p)^{a+b+n}} \, dp$$

$r = 1, 2, 3, ...$

Defined as $B(a,b)$ as the beta function. Defined as $2F1(a,b;c;d)$ as the Gaussian Hypergeometric function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `pGHGBeta` gives the cumulative density values in vector form.

References

Rodriguez-Avi J, Conde-Sanchez A, Saez-Castillo AJ, Olmo-Jimenez MJ (2007). "A generalization of the beta–binomial distribution." *Journal of the Royal Statistical Society Series C: Applied Statistics*, **56**(1), 51–61. Pearson JW (2009). *Computation of hypergeometric functions*. Ph.D. thesis, University of Oxford.

See Also

hypergeo_powerseries

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(.1,.2,.3,1.5,2.15)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,10))
for (i in 1:5)
{
lines(seq(0,1,by=0.001),dGHGBeta(seq(0,1,by=0.001),7,1+a[i],0.3,1+a[i])$pdf,col = col[i])
}

dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$pdf   #extracting the pdf values
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$mean  #extracting the mean
dGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(6)
a <- c(.1,.2,.3,1.5,2.1,3)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:6)
{
lines(seq(0.01,1,by=0.001),pGHGBeta(seq(0.01,1,by=0.001),7,1+a[i],0.3,1+a[i]),col=col[i])
}

pGHGBeta(seq(0,1,by=0.01),7,1.6312,0.3913,0.6659) #acquiring the cumulative probability values
mazGHGBeta(1.4,7,1.6312,0.3913,0.6659)           #acquiring the moment about zero values

#acquiring the variance for a=1.6312,b=0.3913,c=0.6659
mazGHGBeta(2,7,1.6312,0.3913,0.6659)-mazGHGBeta(1,7,1.6312,0.3913,0.6659)^2

#only the integer value of moments is taken here because moments cannot be decimal
mazGHGBeta(1.9,15,5,6,1)
```

---

pGrassiaIIBin *Grassia-II-Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Grassia-II-Binomial Distribution.

Usage

```
pGrassiaIIBin(x,n,a,b)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
a	single value for shape parameter a.
b	single value for shape parameter b.

Details

Mixing Gamma distribution with Binomial distribution will create the the Grassia-II-Binomial distribution, only when (1-p)=e^(-lambda) of the Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{GrassiaIIBin}[x] = \binom{n}{x} \sum_{j=0}^{x} \binom{x}{j} (-1)^{x-j} (1 + b(n-j))^{-a}$$

$$a, b > 0$$

$$x = 0, 1, 2, ..., n$$

$$n = 1, 2, 3, ...$$

The mean, variance and over dispersion are denoted as

$$E_{GrassiaIIBin}[x] = (\frac{b}{b+1})^a$$

$$Var_{GrassiaIIBin}[x] = n^2[(\frac{b}{b+2})^a - (\frac{b}{b+1})^{2a}] + n(\frac{b}{b+1})^a 1 - (\frac{b+1}{b+2})^a$$

$$overdispersion = \frac{(\frac{b}{b+2})^a - (\frac{b}{b+1})^{2a}}{(\frac{b}{b+1})^a[1 - (\frac{b}{b+1})^a]}$$

Value

The output of `pGrassiaIIBin` gives cumulative probability values in vector form.

References

Grassia A (1977). “On a family of distributions with argument between 0 and 1 obtained by transformation of the gamma and derived compound distributions.” *Australian Journal of Statistics*, **19**(2), 108–114.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.3,0.4,0.5,0.6,0.8)
plot(0,0,main="Grassia II binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dGrassiaIIBin(0:10,10,2*a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dGrassiaIIBin(0:10,10,2*a[i],a[i])$pdf,col = col[i],pch=16)
}

dGrassiaIIBin(0:10,10,4,.2)$pdf    #extracting the pdf values
dGrassiaIIBin(0:10,10,4,.2)$mean   #extracting the mean
dGrassiaIIBin(0:10,10,4,.2)$var    #extracting the variance
dGrassiaIIBin(0:10,10,4,.2)$over.dis.para  #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(0.3,0.4,0.5,0.6)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pGrassiaIIBin(0:10,10,2*a[i],a[i]),col = col[i])
points(0:10,pGrassiaIIBin(0:10,10,2*a[i],a[i]),col = col[i])
}

pGrassiaIIBin(0:10,10,4,.2)   #acquiring the cumulative probability values
```

---

pKUM *Kumaraswamy Distribution*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moment about zero values for the Kumaraswamy Distribution bounded between [0,1].

Usage

```
pKUM(p,a,b)
```

Arguments

p	vector of probabilities.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.

Details

The probability density function and cumulative density function of a unit bounded Kumaraswamy Distribution with random variable P are given by

$$g_P(p) = abp^{a-1}(1-p^a)^{b-1}$$

; $0 \le p \le 1$

$$G_P(p) = 1 - (1-p^a)^b$$

; $0 \le p \le 1$

$$a, b > 0$$

The mean and the variance are denoted by

$$E[P] = bB(1 + \frac{1}{a}, b)$$

$$var[P] = bB(1 + \frac{2}{a}, b) - (bB(1 + \frac{1}{a}, b))^2$$

The moments about zero is denoted as

$$E[P^r] = bB(1 + \frac{r}{a}, b)$$

$r = 1, 2, 3, ...$

Defined as $B(a, b)$ is the beta function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `pKUM` gives the cumulative density values in vector form.

References

Kumaraswamy P (1980). "A generalized probability density function for double-bounded random processes." *Journal of hydrology*, **46**(1-2), 79–88. Jones MC (2009). "Kumaraswamy's distribution: A beta-type distribution with some tractability advantages." *Statistical methodology*, **6**(1), 70–81.

Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Probability density graph",xlab="Random variable",ylab="Probability density values",
xlim = c(0,1),ylim = c(0,6))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dKUM(seq(0,1,by=0.01),a[i],a[i])$pdf,col = col[i])
}
```

```
dKUM(seq(0,1,by=0.01),2,3)$pdf   #extracting the probability values
dKUM(seq(0,1,by=0.01),2,3)$mean  #extracting the mean
dKUM(seq(0,1,by=0.01),2,3)$var   #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative density graph",xlab="Random variable",ylab="Cumulative density values",
xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pKUM(seq(0,1,by=0.01),a[i],a[i]),col = col[i])
}

pKUM(seq(0,1,by=0.01),2,3)    #acquiring the cumulative probability values

mazKUM(1.4,3,2)               #acquiring the moment about zero values
mazKUM(2,2,3)-mazKUM(1,2,3)^2  #acquiring the variance for a=2,b=3

#only the integer value of moments is taken here because moments cannot be decimal
mazKUM(1.9,5.5,6)
```

---

pKumBin *Kumaraswamy Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Kumaraswamy Binomial Distribution.

Usage

```
pKumBin(x,n,a,b,it=25000)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trial.
a	single value for shape parameter alpha representing a.
b	single value for shape parameter beta representing b.
it	number of iterations to converge as a proper probability function replacing infinity.

Details

Mixing Kumaraswamy distribution with Binomial distribution will create the Kumaraswamy Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{KumBin}(x) = ab\binom{n}{x}\sum_{j=0}^{it}(-1)^j\binom{b-1}{j}B(x+a+aj, n-x+1)$$

$$a, b > 0$$

$$x = 0, 1, 2, ...n$$

$$n = 1, 2, 3, ...$$

$$it > 0$$

The mean, variance and over dispersion are denoted as

$$E_{KumBin}[x] = nbB(1+\frac{1}{a}, b)$$

$$Var_{KumBin}[x] = (n^2)b(B(1+\frac{2}{a}, b) - bB(1+\frac{1}{a}, b)^2) + nb(B(1+\frac{1}{a}, b) - B(1+\frac{2}{a}, b))$$

$$overdispersion = \frac{(bB(1+\frac{2}{a}, b) - (bB(1+\frac{1}{a}, b))^2)}{(bB(1+\frac{1}{a}, b) - (bB(1+\frac{1}{a}, b))^2)}$$

Defined as $B(a, b)$ is the beta function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `pKumBin` gives cumulative probability values in vector form.

References

Xiaohu L, Yanyan H, Xueyan Z (2011). "The Kumaraswamy binomial distribution." *Chinese Journal of Applied Probability and Statistics*, **27**(5), 511–521.

Examples

```
## Not run:
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,.85)
plot(0,0,main="Kumaraswamy binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5) {
lines(0:10,dKumBin(0:10,10,a[i],a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dKumBin(0:10,10,a[i],a[i])$pdf,col = col[i],pch=16)
```

```
  }

## End(Not run)

dKumBin(0:10,10,4,2)$pdf  #extracting the pdf values
dKumBin(0:10,10,4,2)$mean #extracting the mean
dKumBin(0:10,10,4,2)$var  #extracting the variance
dKumBin(0:10,10,4,2)$over.dis.para #extracting the over dispersion value

## Not run:
#plotting the random variables and cumulative probability values
col <- rainbow(5)
a <- c(1,2,5,10,.85)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5) {
lines(0:10,pKumBin(0:10,10,a[i],a[i]),col = col[i])
points(0:10,pKumBin(0:10,10,a[i],a[i]),col = col[i])
  }

## End(Not run)
pKumBin(0:10,10,4,2)    #acquiring the cumulative probability values
```

`Plant_DiseaseData` *Plant Disease Incidence data*

Description

Cochran(1936) provided a data that comprise the number of tomato spotted wilt virus(TSWV) infected tomato plants in the field trials in Australia. The field map was divided into 160 'quadrats'. 9 tomato plants in each quadrat. then the numbers of TSWV infected tomato plants were counted in each quadrat. Number of infected plants out of 9 plants per quadrat can be treated as a binomial variable. the collection of all such responses from all 160 quadrats would form "binomial outcome data" below provided is a data set similar to Cochran plant disease incidence data. Marcus R(1984). orange trees infected with citrus tristeza virus (CTV) in an orchard in central Israel. We divided the field map into 84 "quadrats" of 4 rows x 3 columns and counted the total number (1981 + 1982) of infected trees out of a maximum of n = 12 in each quadrat

Usage

```
Plant_DiseaseData
```

Format

A data frame with 2 columns and 10 rows

`Dis.plant` Diseased Plants

`fre` Observed frequencies

Source

Extracted from

Hughes, G., 1993. Using the Beta-Binomial Distribution to Describe Aggregated Patterns of Disease Incidence. Phytopathology, 83(9), p.759.

Available at: doi:10.1094/Phyto83759.

Examples

```
Plant_DiseaseData$Dis.plant      # extracting the binomial random variables
sum(Plant_DiseaseData$fre)       # summing all the frequencies
```

---

pLMBin *Lovinson Multiplicative Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Lovinson Multiplicative Binomial Distribution.

Usage

```
pLMBin(x,n,p,phi)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
p	single value for probability of success.
phi	single value for phi.

Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{LMBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} \frac{(phi^{x(n-x)}}{f(p, phi, n)}$$

here $f(p, phi, n)$ is

$$f(p, phi, n) = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} (phi^{k(n-k)})$$

$$x = 0, 1, 2, 3, ...n$$

$$n = 1, 2, 3, ...$$
$$k = 0, 1, 2, ..., n$$
$$0 < p < 1$$
$$0 < phi$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `pLMBin` gives cumulative probability values in vector form.

References

Elamir EA (2013). "Multiplicative-Binomial Distribution: Some Results on Characterization, Inference and Random Data Generation." *Journal of Statistical Theory and Applications*, **12**(1), 92–105.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Lovinson Multiplicative binomial probability
     function graph",xlab="Binomial random variable",
     ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dLMBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dLMBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],pch=16)
}

dLMBin(0:10,10,.58,10.022)$pdf   #extracting the pdf values
dLMBin(0:10,10,.58,10.022)$mean   #extracting the mean
dLMBin(0:10,10,.58,10.022)$var   #extracting the variance

#plotting random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Lovinson Multiplicative binomial probability
     function graph",xlab="Binomial random variable",
     ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pLMBin(0:10,10,a[i],1+b[i]),col = col[i],lwd=2.85)
points(0:10,pLMBin(0:10,10,a[i],1+b[i]),col = col[i],pch=16)
}

pLMBin(0:10,10,.58,10.022)     #acquiring the cumulative probability values
```

pMcGBB *McDonald Generalized Beta Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the McDonald Generalized Beta Binomial Distribution.

Usage

```
pMcGBB(x,n,a,b,c)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.
a	single value for shape parameter alpha representing as a.
b	single value for shape parameter beta representing as b.
c	single value for shape parameter gamma representing as c.

Details

Mixing Generalized Beta Type-1 Distribution with Binomial distribution the probability function value and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{McGBB}(x) = \binom{n}{x} \frac{1}{B(a,b)} (\sum_{j=0}^{n-x} (-1)^j \binom{n-x}{j} B(\frac{x}{c} + a + \frac{j}{c}, b))$$

$$a, b, c > 0$$

The mean, variance and over dispersion are denoted as

$$E_{McGBB}[x] = n \frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})}$$

$$Var_{McGBB}[x] = n^2 (\frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - (\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})})^2) + n(\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} - \frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})})$$

$$overdispersion = \frac{\frac{B(a+b, \frac{2}{c})}{B(a, \frac{2}{c})} - (\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})})^2}{\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})} - (\frac{B(a+b, \frac{1}{c})}{B(a, \frac{1}{c})})^2}$$

$$x = 0, 1, 2, ...n$$

$$n = 1, 2, 3, ...$$

Value

The output of pMcGBB gives cumulative probability function values in vector form.

References

Manoj C, Wijekoon P, Yapa RD (2013). "The McDonald generalized beta-binomial distribution: A new binomial mixture distribution and simulation based comparison with its nested distributions in handling overdispersion." *International journal of statistics and probability*, **2**(2), 24. Janiffer NM, Islam A, Luke O, others (2014). "Estimating Equations for Estimation of Mcdonald Generalized Beta—Binomial Parameters." *Open Journal of Statistics*, **4**(09), 702. Roozegar R, Tahmasebi S, Jafari AA (2017). "The McDonald Gompertz distribution: properties and applications." *Communications in Statistics-Simulation and Computation*, **46**(5), 3341–3355.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(1,2,5,10,0.6)
plot(0,0,main="Mcdonald generalized beta-binomial probability function graph",
xlab="Binomial random variable",ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dMcGBB(0:10,10,a[i],2.5,a[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dMcGBB(0:10,10,a[i],2.5,a[i])$pdf,col = col[i],pch=16)
}

dMcGBB(0:10,10,4,2,1)$pdf             #extracting the pdf values
dMcGBB(0:10,10,4,2,1)$mean            #extracting the mean
dMcGBB(0:10,10,4,2,1)$var             #extracting the variance
dMcGBB(0:10,10,4,2,1)$over.dis.para   #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(4)
a <- c(1,2,5,10)
plot(0,0,main="Cumulative probability function graph",xlab="Binomial random variable",
ylab="Cumulative probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:4)
{
lines(0:10,pMcGBB(0:10,10,a[i],a[i],2),col = col[i])
points(0:10,pMcGBB(0:10,10,a[i],a[i],2),col = col[i])
}

pMcGBB(0:10,10,4,2,1)       #acquiring the cumulative probability values
```

---

pMultiBin *Multiplicative Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Multiplicative Binomial Distribution.

Usage

```
pMultiBin(x,n,p,theta)
```

Arguments

`x`	vector of binomial random variables.
`n`	single value for no of binomial trials.
`p`	single value for probability of success.
`theta`	single value for theta.

Details

The probability function and cumulative function can be constructed and are denoted below

The cumulative probability function is the summation of probability function values.

$$P_{MultiBin}(x) = \binom{n}{x} p^x (1-p)^{n-x} \frac{(theta^{x(n-x)}}{f(p,theta,n)}$$

here $f(p,theta,n)$ is

$$f(p,theta,n) = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} (theta^{k(n-k)})$$

$$x = 0,1,2,3,...n$$
$$n = 1,2,3,...$$
$$k = 0,1,2,...,n$$
$$0 < p < 1$$
$$0 < theta$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `pMultiBin` gives cumulative probability values in vector form.

References

Johnson NL, Kemp AW, Kotz S (2005). *Univariate discrete distributions*, volume 444. John Wiley and Sons. Kupper LL, Haseman JK (1978). "The use of a correlated binomial model for the analysis of certain toxicological experiments." *Biometrics*, 69–76. Paul SR (1985). "A three-parameter generalization of the binomial distribution." *History and Philosophy of Logic*, **14**(6), 1497–1506.

Examples

```
#plotting the random variables and probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Multiplicative binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,0.5))
for (i in 1:5)
{
lines(0:10,dMultiBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dMultiBin(0:10,10,a[i],1+b[i])$pdf,col = col[i],pch=16)
}

dMultiBin(0:10,10,.58,10.022)$pdf   #extracting the pdf values
dMultiBin(0:10,10,.58,10.022)$mean   #extracting the mean
dMultiBin(0:10,10,.58,10.022)$var   #extracting the variance

#plotting random variables and cumulative probability values
col <- rainbow(5)
a <- c(0.58,0.59,0.6,0.61,0.62)
b <- c(0.022,0.023,0.024,0.025,0.026)
plot(0,0,main="Multiplicative binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:5)
{
lines(0:10,pMultiBin(0:10,10,a[i],1+b[i]),col = col[i],lwd=2.85)
points(0:10,pMultiBin(0:10,10,a[i],1+b[i]),col = col[i],pch=16)
}

pMultiBin(0:10,10,.58,10.022)     #acquiring the cumulative probability values
```

---

pTRI *Triangular Distribution Bounded Between [0,1]*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Triangular Distribution bounded between [0,1].

Usage

```
pTRI(p,mode)
```

Arguments

p	vector of probabilities.
mode	single value for mode.

Details

Setting $min = 0$ and $max = 1$ $mode = c$ in the Triangular distribution a unit bounded Triangular distribution can be obtained. The probability density function and cumulative density function of a unit bounded Triangular distribution with random variable P are given by

$$g_P(p) = \frac{2p}{c}$$

; $0 \le p < c$

$$g_P(p) = \frac{2(1-p)}{(1-c)}$$

; $c \le p \le 1$

$$G_P(p) = \frac{p^2}{c}$$

; $0 \le p < c$

$$G_P(p) = 1 - \frac{(1-p)^2}{(1-c)}$$

; $c \le p \le 1$

$$0 \le mode = c \le 1$$

The mean and the variance are denoted by

$$E[P] = \frac{(a+b+c)}{3} = \frac{(1+c)}{3}$$

$$var[P] = \frac{a^2+b^2+c^2-ab-ac-bc}{18} = \frac{(1+c^2-c)}{18}$$

Moments about zero is denoted as

$$E[P^r] = \frac{2c^{r+2}}{c(r+2)} + \frac{2(1-c^{r+1})}{(1-c)(r+1)} + \frac{2(c^{r+2}-1)}{(1-c)(r+2)}$$

$r = 1, 2, 3, ...$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `pTRI` gives the cumulative density values in vector form.

References

Horsnell G (1957). "Economical acceptance sampling schemes." *Journal of the Royal Statistical Society. Series A (General)*, **120**(2), 148–201. Johnson NL, Kotz S, Balakrishnan N (1995). *Continuous univariate distributions, volume 2*, volume 289. John wiley and sons. Karlis D, Xekalaki E (2008). *The polygonal distribution*. Springer. Okagbue HI, Edeki SO, Opanuga AA, Oguntunde PE, Adeosun ME (2014). "Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution." *British Journal of Mathematics and Computer Science*, **4**(24), 3497.

Examples

```
#plotting the random variables and probability values
col <- rainbow(4)
x <- seq(0.2,0.8,by=0.2)
plot(0,0,main="Probability density graph",xlab="Random variable",
ylab="Probability density values",xlim = c(0,1),ylim = c(0,3))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),dTRI(seq(0,1,by=0.01),x[i])$pdf,col = col[i])
}

dTRI(seq(0,1,by=0.05),0.3)$pdf     #extracting the pdf values
dTRI(seq(0,1,by=0.01),0.3)$mean    #extracting the mean
dTRI(seq(0,1,by=0.01),0.3)$var     #extracting the variance

#plotting the random variables and cumulative probability values
col <- rainbow(4)
x <- seq(0.2,0.8,by=0.2)
plot(0,0,main="Cumulative density graph",xlab="Random variable",
ylab="Cumulative density values",xlim = c(0,1),ylim = c(0,1))
for (i in 1:4)
{
lines(seq(0,1,by=0.01),pTRI(seq(0,1,by=0.01),x[i]),col = col[i])
}

pTRI(seq(0,1,by=0.05),0.3)      #acquiring the cumulative probability values
mazTRI(1.4,.3)                  #acquiring the moment about zero values
mazTRI(2,.3)-mazTRI(1,.3)^2     #variance for when is mode 0.3

#only the integer value of moments is taken here because moments cannot be decimal
mazTRI(1.9,0.5)
```

---

pTriBin *Triangular Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Triangular Binomial distribution.

Usage

```
pTriBin(x,n,mode)
```

Arguments

x	vector of binomial random variables
n	single value for no of binomial trials
mode	single value for mode

Details

Mixing unit bounded Triangular distribution with Binomial distribution will create Triangular Binomial distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{TriBin}(x) = 2\binom{n}{x}(c^{-1}B_c(x+2,n-x+1)+(1-c)^{-1}B(x+1,n-x+2)-(1-c)^{-1}B_c(x+1,n-x+2))$$

$$0 < mode = c < 1$$

$$x = 0, 1, 2, ...n$$

$$n = 1, 2, 3...$$

The mean, variance and over dispersion are denoted as

$$E_{TriiBin}[x] = \frac{n(1+c)}{3}$$

$$Var_{TriBin}[x] = \frac{n(n+3)}{18} - \frac{n(n-3)c(1-c)}{18}$$

$$overdispersion = \frac{(1-c+c^2)}{2(2+c-c^2)}$$

Defined as $B_c(a,b) = \int_0^c t^{a-1}(1-t)^{b-1}\,dt$ is incomplete beta integrals and $B(a,b)$ is the beta function.

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `pTriBin` gives cumulative probability function values in vector form.

References

Horsnell G (1957). “Economical acceptance sampling schemes.” *Journal of the Royal Statistical Society. Series A (General)*, **120**(2), 148–201. Karlis D, Xekalaki E (2008). *The polygonal distribution*. Springer. Okagbue HI, Edeki SO, Opanuga AA, Oguntunde PE, Adeosun ME (2014). “Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution.” *British Journal of Mathematics and Computer Science*, **4**(24), 3497.

Examples

```
#plotting the random variables and probability values
col <- rainbow(7)
x <- seq(0.1,0.7,by=0.1)
plot(0,0,main="Triangular binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,.3))
for (i in 1:7)
{
lines(0:10,dTriBin(0:10,10,x[i])$pdf,col = col[i],lwd=2.85)
points(0:10,dTriBin(0:10,10,x[i])$pdf,col = col[i],pch=16)
}

dTriBin(0:10,10,.4)$pdf        #extracting the pdf values
dTriBin(0:10,10,.4)$mean       #extracting the mean
dTriBin(0:10,10,.4)$var        #extracting the variance
dTriBin(0:10,10,.4)$over.dis.para  #extracting the over dispersion value

#plotting the random variables and cumulative probability values
col <- rainbow(7)
x <- seq(0.1,0.7,by=0.1)
plot(0,0,main="Triangular binomial probability function graph",xlab="Binomial random variable",
ylab="Probability function values",xlim = c(0,10),ylim = c(0,1))
for (i in 1:7)
{
lines(0:10,pTriBin(0:10,10,x[i]),col = col[i],lwd=2.85)
points(0:10,pTriBin(0:10,10,x[i]),col = col[i],pch=16)
}

pTriBin(0:10,10,.4)    #acquiring the cumulative probability values
```

---

pUNI *Uniform Distribution Bounded Between [0,1]*

Description

These functions provide the ability for generating probability density values, cumulative probability density values and moments about zero values for the Uniform Distribution bounded between [0,1].

Usage

```
pUNI(p)
```

Arguments

p	vector of probabilities.

Details

Setting $a = 0$ and $b = 1$ in the Uniform Distribution a unit bounded Uniform Distribution can be obtained. The probability density function and cumulative density function of a unit bounded Uniform Distribution with random variable P are given by

$$g_P(p) = 1$$

$0 \leq p \leq 1$

$$G_P(p) = p$$

$0 \leq p \leq 1$

The mean and the variance are denoted as

$$E[P] = \frac{1}{a+b} = 0.5$$

$$var[P] = \frac{(b-a)^2}{12} = 0.0833$$

Moments about zero is denoted as

$$E[P^r] = \frac{e^{rb} - e^{ra}}{r(b-a)} = \frac{e^r - 1}{r}$$

$r = 1, 2, 3, ...$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `pUNI` gives the cumulative density values in vector form.

References

Horsnell G (1957). "Economical acceptance sampling schemes." *Journal of the Royal Statistical Society. Series A (General)*, **120**(2), 148–201. Johnson NL, Kotz S, Balakrishnan N (1995). *Continuous univariate distributions, volume 2*, volume 289. John wiley and sons.

See Also

`Uniform`

or

https://stat.ethz.ch/R-manual/R-devel/library/stats/html/Uniform.html

Examples

```
#plotting the random variables and probability values
plot(seq(0,1,by=0.01),dUNI(seq(0,1,by=0.01))$pdf,type = "l",main="Probability density graph",
xlab="Random variable",ylab="Probability density values")

dUNI(seq(0,1,by=0.05))$pdf     #extract the pdf values
dUNI(seq(0,1,by=0.01))$mean    #extract the mean
dUNI(seq(0,1,by=0.01))$var     #extract the variance

#plotting the random variables and cumulative probability values
plot(seq(0,1,by=0.01),pUNI(seq(0,1,by=0.01)),type = "l",main="Cumulative density graph",
xlab="Random variable",ylab="Cumulative density values")

pUNI(seq(0,1,by=0.05))     #acquiring the cumulative probability values

mazUNI(c(1,2,3))    #acquiring the moment about zero values

#only the integer value of moments is taken here because moments cannot be decimal
mazUNI(1.9)
```

---

pUniBin *Uniform Binomial Distribution*

Description

These functions provide the ability for generating probability function values and cumulative probability function values for the Uniform Binomial Distribution.

Usage

```
pUniBin(x,n)
```

Arguments

x	vector of binomial random variables.
n	single value for no of binomial trials.

Details

Mixing unit bounded uniform distribution with binomial distribution will create the Uniform Binomial Distribution. The probability function and cumulative probability function can be constructed and are denoted below.

The cumulative probability function is the summation of probability function values.

$$P_{UniBin}(x) = \frac{1}{n+1}$$
$$n = 1, 2, ...$$

$$x = 0, 1, 2, ...n$$

The mean, variance and over dispersion are denoted as

$$E_{UniBin}[X] = \frac{n}{2}$$

$$Var_{UniBin}[X] = \frac{n(n+2)}{12}$$

$$overdispersion = \frac{1}{3}$$

NOTE : If input parameters are not in given domain conditions necessary error messages will be provided to go further.

Value

The output of `pUniBin` gives cumulative probability function values in vector form.

References

Horsnell G (1957). "Economical acceptance sampling schemes." *Journal of the Royal Statistical Society. Series A (General)*, **120**(2), 148–201. Okagbue HI, Edeki SO, Opanuga AA, Oguntunde PE, Adeosun ME (2014). "Using the Average of the Extreme Values of a Triangular Distribution for a Transformation, and Its Approximant via the Continuous Uniform Distribution." *British Journal of Mathematics and Computer Science*, **4**(24), 3497.

Examples

```
#plotting the binomial random variables and probability values
plot(0:10,dUniBin(0:10,10)$pdf,type="l",main="Uniform binomial probability function graph",
xlab=" Binomial random variable",ylab="Probability function values")
points(0:10,dUniBin(0:10,10)$pdf)

dUniBin(0:300,300)$pdf  #extracting the pdf values
dUniBin(0:10,10)$mean   #extracting the mean
dUniBin(0:10,10)$var    #extracting the variance
dUniBin(0:10,10)$over.dis.para  #extracting the over dispersion

#plotting the binomial random variables and cumulative probability values
plot(0:10,pUniBin(0:10,10),type="l",main="Cumulative probability function graph",
xlab=" Binomial random variable",ylab="Cumulative probability function values")
points(0:10,pUniBin(0:10,10))

pUniBin(0:15,15)       #acquiring the cumulative probability values
```

`Terror_data_ARG` *Terror Data ARG*

Description

Jenkins and Johnson (1975) compiled a chronology of incidents of international terrorism from 1/1968 through 04/1974. During this period 507 incidents are recorded in the world, where 64 incidents occurred in the United States and 65 ones in Argentina.

Usage

```
Terror_data_ARG
```

Format

A data frame with 2 columns and 9 rows

`Incidents` No of Incidents Occurred

`fre` Observed frequencies

Source

Extracted from

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

Examples

```
Terror_data_ARG$Incidents        #extracting the binomial random variables
sum(Terror_data_ARG$fre)              #summing all the frequencies
```

`Terror_data_USA` *Terror Data USA*

Description

Jenkins and Johnson (1975) compiled a chronology of incidents of international terrorism from 1/1968 through 04/1974. During this period 507 incidents are recorded in the world, where 64 incidents occurred in the United States and 65 ones in Argentina.

Usage

```
Terror_data_USA
```

Format

A data frame with 2 columns and 9 rows

`Incidents` No of Incidents Occurred

`fre` Observed frequencies

Source

Extracted from

Li, X. H., Huang, Y. Y., & Zhao, X. Y. (2011). The Kumaraswamy Binomial Distribution. Chinese Journal of Applied Probability and Statistics, 27(5), 511-521.

Examples

```
Terror_data_USA$Incidents         #extracting the binomial random variables
sum(Terror_data_USA$fre)              #summing all the frequencies
```

Index