

# Package ‘lsirm12pl’

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**Type** Package

**Title** Latent Space Item Response Model

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**Description** Analysis of dichotomous and continuous response data using latent factor by both 1PL LSIRM and 2PL LSIRM as described in Jeon et al. (2021) <[doi:10.1007/s11336-021-09762-5](https://doi.org/10.1007/s11336-021-09762-5)>. It includes original 1PL LSIRM and 2PL LSIRM provided for binary response data and its extension for continuous response data. Bayesian model selection with spike-and-slab prior and method for dealing data with missing value under missing at random, missing completely at random are also supported. Various diagnostic plots are available to inspect the latent space and summary of estimated parameters.

**License** GPL-3

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*BFPT* *Big Five Personality Test*

---

### Description

A dataset containing the result of personality test for 50 questions from 1,000 random sampled people.

### Usage

```
data(BFPT)
```

### Format

A matrix with 1,015,341 rows and 50 columns.

### Details

A dataset collected in 2016-2018 through an interactive on-line personality test, containing the result of personality test for 50 questions. 1,000 people are random sampled from the original dataset containing 1,015,341 people. The scale is labeled as 1=Disagree, 3=Neutral and 5=Agree.

### Source

<https://www.kaggle.com/tunguz/big-five-personality-test>

---

*diagnostic* *Diagnostic the result of LSIRM.*

---

### Description

`diagnostic` checks the convergence of MCMC for LSIRM parameters using various diagnostic tools, such as trace plots, posterior density distributions, autocorrelation functions (ACF), and Gelman-Rubin-Brooks plots.

### Usage

```
diagnostic(  
  object,  
  draw.item = list(beta = c(1), theta = c(1)),  
  gelman.diag = FALSE  
)
```

**Arguments**

object	Object of class <code>lsirm</code> .
draw.item	List; Each key in the list corresponds to a specific parameters such as "beta", "theta", "gamma", "alpha", "sigma", "sigma_sd", and "zw.dist". The values of the list indicate the indices of these parameters. For the key "zw.dist", the value is a matrix with two columns: the first column represents the indices of respondents, and the second column represents the indices of items.
gelman.diag	Logical; If TRUE, the Gelman-Rubin convergence diagnostic will be printed. Default is FALSE.

**Value**

diagnostic returns plots for checking MCMC convergence for selected parameters.

**Examples**

```
# Generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

# For 1PL LSIRM
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = FALSE))
diagnostic(lsirm_result)

# For 2PL LSIRM
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = FALSE))
diagnostic(lsirm_result)
```

---

gof

*Goodness-of-fit LSIRM*


---

**Description**

`gof` is goodness-of-fit the latent space of fitted LSIRM.

**Usage**

```
gof(object, chain.idx = 1)
```

**Arguments**

object	Object of class <code>lsirm</code> .
chain.idx	Numeric; Index of MCMC chain. Default is 1.

**Value**

`gof` returns the boxplot or AUC plot

## Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)
lsirm_result <- lsirm(data ~ lsirm1pl())
gof(lsirm_result)
```

---

lsirm

*Fit a LSIRM ( Latent Space Item Response Model)*

---

## Description

[lsirm](#) is used to fit 1PL LSIRM and 2PL LSIRM using Bayesian method as described in Jeon et al. (2021).

## Usage

```
lsirm(formula, ...)
```

## Arguments

formula	The form of formula is <code>lsirm(A ~ &lt;term 1&gt;(&lt;term 2&gt;, &lt;term 3&gt; ...))</code> , where A is binary or continuous item response matrix to be analyzed, <term1> is the model you want to fit and has one of the following values: "lsirm1pl" and "lsirm2pl"., and <term 2>, <term 3>, etc. are each option for the model.
...	Additional arguments for the corresponding function.

## Details

The descriptions of options for each model, such as <term 2> and <term 3>, are included in [lsirm1pl](#) for 1PL LSIRM and [lsirm2pl](#) for 2PL LSIRM.

## Value

`lsirm` returns an object of class `list`.

See corresponding functions such as [lsirm1pl](#) for 1PL LSIRM and [lsirm2pl](#) for 2PL LSIRM.

## See Also

[lsirm1pl](#) for 1PL LSIRM.

[lsirm2pl](#) for 2PL LSIRM.

**Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

lsirm_result <- lsirm(data~lsirm1pl())
lsirm_result <- lsirm(data~lsirm2pl())
```

---

lsirm.formula	<i>Formula function for LSIRM</i>
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---

**Description**

`lsirm.formula` is formula object.

**Usage**

```
## S3 method for class 'formula'
lsirm(formula, ...)
```

**Arguments**

formula	The form of formula is <code>lsirm(A ~ &lt;term 1&gt;(&lt;term 2&gt;, &lt;term 3&gt; ...))</code> , where A is binary or continuous item response matrix to be analyzed, <term1> is the model you want to fit and has one of the following values: "lsirm1pl" and "lsirm2pl"., and <term 2>, <term 3>, etc., are each option for the model.
...	Additional arguments for the corresponding function.

---

lsirm12pl	<i>lsirm12pl-package</i>
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---

**Description**

Analysis of dichotomous and continuous response data using latent factor by both 1PL LSIRM and 2PL LSIRM as described in Jeon et al. (2021) <doi:10.1007/s11336-021-09762-5>. It includes original 1PL LSIRM and 2PL LSIRM provided for binary response data and its extension for continuous response data. Bayesian model selection with spike-and-slab prior and method for dealing data with missing value under missing at random, missing completely at random are also supported. Various diagnostic plots are available to inspect the latent space and summary of estimated parameters.

---

`lsirm1pl`*Fit a 1PL LSIRM for binary and continuous item response data*

---

## Description

`lsirm1pl` integrates all functions related to 1PL LSIRM. Various 1PL LSIRM function can be used by setting the `spikenslab`, `fixed_gamma`, and `missing_data` arguments.

This function can be used regardless of the data type, providing a unified approach to model fitting.

## Usage

```
lsirm1pl(  
  data,  
  spikenslab = FALSE,  
  fixed_gamma = FALSE,  
  missing_data = NA,  
  chains = 1,  
  multicore = 1,  
  seed = NA,  
  ndim = 2,  
  niter = 15000,  
  nburn = 2500,  
  nthin = 5,  
  nprint = 500,  
  jump_beta = 0.4,  
  jump_theta = 1,  
  jump_z = 0.5,  
  jump_w = 0.5,  
  pr_mean_beta = 0,  
  pr_sd_beta = 1,  
  pr_mean_theta = 0,  
  pr_a_theta = 0.001,  
  pr_b_theta = 0.001,  
  ...  
)
```

## Arguments

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>spikenslab</code>	Logical; specifies whether to use a model selection approach. Default is FALSE.
<code>fixed_gamma</code>	Logical; indicates whether to fix gamma at 1. Default is FALSE.
<code>missing_data</code>	Character; the type of missing data assumed. Options are NA, "mar", or "mcar". Default is NA.

chains	Integer; the number of MCMC chains to run. Default is 1.
multicore	Integer; the number of cores to use for parallel execution. Default is 1.
seed	Integer; the seed number for MCMC fitting. Default is NA.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
...	Additional arguments for the for various settings. Refer to the functions in the Details.

## Details

Additional arguments and return values for each function are documented in the respective function's description.

\* For LSIRM with data included missing value are detailed in [lsirm1pl\\_mar](#) and [lsirm1pl\\_mcar](#).

\* For LSIRM using the spike-and-slab model selection approach are detailed in [lsirm1pl\\_ss](#).

\* For continuous version of LSIRM are detailed in [lsirm1pl\\_normal\\_o](#).

For 1PL LSIRM with binary item response data, the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$\text{logit}(P(Y_{j,i} = 1 | \theta_j, \beta_i, \gamma, z_j, w_i)) = \theta_j + \beta_i - \gamma ||z_j - w_i||$$

For 1PL LSIRM with continuous item response data, the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ .



**Value**

lsirm1pl returns an object of list. The basic return list containing the following components:

data	A data frame or matrix containing the variables used in the model.
bic	A numeric value representing the Bayesian Information Criterion (BIC).
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
...	Additional return values for various settings. Refer to the functions in the Details.

**Note**

If both `spikenslab` and `fixed_gamma` are set TRUE, it returns error because both are related to `gamma`.

**See Also**

The LSIRM for 1PL LSIRM for binary item response data as following:

[lsirm1pl\\_o](#), [lsirm1pl\\_fixed\\_gamma](#), [lsirm1pl\\_mar](#), [lsirm1pl\\_mcar](#), [lsirm1pl\\_fixed\\_gamma\\_mar](#), [lsirm1pl\\_fixed\\_gamma\\_mcar](#), [lsirm1pl\\_ss](#), [lsirm1pl\\_mar\\_ss](#), and [lsirm1pl\\_mcar\\_ss](#)

The LSIRM for 1PL LSIRM for continuous item response data as following:

[lsirm1pl\\_normal\\_o](#), [lsirm1pl\\_normal\\_fixed\\_gamma](#), [lsirm1pl\\_normal\\_mar](#), [lsirm1pl\\_normal\\_mcar](#), [lsirm1pl\\_normal\\_mar\\_ss](#), [lsirm1pl\\_normal\\_mcar\\_ss](#)

**Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)
lsirm_result <- lsirm1pl(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data~lsirm1pl())
```

---

lsirm1pl\_fixed\_gamma *1PL LSIRM fixing gamma to 1.*

---

**Description**

[lsirm1pl\\_fixed\\_gamma](#) is used to fit 1PL LSIRM with gamma fixed to 1. [lsirm1pl\\_fixed\\_gamma](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. The resulting latent space provides an interaction map that represents interactions between respondents and items.

**Usage**

```
lsirm1pl_fixed_gamma(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  verbose = FALSE
)
```

**Arguments**

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.

niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. default value is FALSE

## Details

lsirm1pl\_fixed\_gamma models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space:

$$\text{logit}(P(Y_{j,i} = 1 | \theta_j, \beta_i, z_j, w_i)) = \theta_j + \beta_i - \|z_j - w_i\|$$

## Value

lsirm1pl\_fixed\_gamma returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
z_estimate	Posterior estimates of the z parameter.

w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.

### Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

lsirm_result <- lsirm1pl_fixed_gamma(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = TRUE))
```

---

```
lsirm1pl_fixed_gamma_mar
```

*1PL LSIRM fixing gamma to 1 for missing at random data.*

---

### Description

[lsirm1pl\\_fixed\\_gamma\\_mar](#) is used to fit LSIRM with gamma fixed to 1 in incomplete data assumed to be missing at random. [lsirm1pl\\_fixed\\_gamma\\_mar](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm1pl_fixed_gamma_mar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
```

```

    jump_beta = 0.4,
    jump_theta = 1,
    jump_z = 0.5,
    jump_w = 0.5,
    pr_mean_beta = 0,
    pr_sd_beta = 1,
    pr_mean_theta = 0,
    pr_a_theta = 0.001,
    pr_b_theta = 0.001,
    missing.val = 99,
    verbose = FALSE
)

```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. default value is FALSE.

## Details

lsirm1pl\_fixed\_gamma\_mar models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space:

$$\text{logit}(P(Y_{j,i} = 1 | \theta_j, \beta_i, z_j, w_i)) = \theta_j + \beta_i - \|z_j - w_i\|$$

Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

## Value

lsirm1pl\_fixed\_gamma\_mar returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
imp_estimate	Probability of imputating a missing value with 1.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
imp	Imputation for missing Values using posterior samples.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

## Examples

```
# generate example item response matrix
data      <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat  <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_fixed_gamma_mar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = TRUE,
missing_data = "mar", missing.val = 99))
```

---

```
lsirm1pl_fixed_gamma_mcar
```

*1PL LSIRM fixing gamma to 1 for missing completely at random data.*

---

## Description

`lsirm1pl_fixed_gamma_mcar` is used to fit LSIRM with gamma fixed to 1 in incomplete data assumed to be missing completely at random. `lsirm1pl_fixed_gamma_mcar` factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm1pl_fixed_gamma_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_z = 0.5,
  jump_w = 0.5,
```

```

pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
missing.val = 99,
verbose = FALSE
)

```

### Arguments

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>ndim</code>	Integer; the dimension of the latent space. Default is 2.
<code>niter</code>	Integer; the total number of MCMC iterations to run. Default is 15000.
<code>nburn</code>	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
<code>nthin</code>	Integer; the number of MCMC iterations to thin. Default is 5.
<code>nprint</code>	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
<code>jump_beta</code>	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
<code>jump_theta</code>	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
<code>jump_z</code>	Numeric; the jumping rule for the z proposal density. Default is 0.5.
<code>jump_w</code>	Numeric; the jumping rule for the w proposal density. Default is 0.5.
<code>pr_mean_beta</code>	Numeric; the mean of the normal prior for beta. Default is 0.
<code>pr_sd_beta</code>	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
<code>pr_mean_theta</code>	Numeric; the mean of the normal prior for theta. Default is 0.
<code>pr_a_theta</code>	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
<code>pr_b_theta</code>	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
<code>missing.val</code>	Numeric; a number to replace missing values. Default is 99.
<code>verbose</code>	Logical; If TRUE, MCMC samples are printed for each <code>nprint</code> . default value is FALSE

### Details

`lsirm1pl_fixed_gamma_mcar` models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space:

$$\text{logit}(P(Y_{j,i} = 1 | \theta_j, \beta_i, z_j, w_i)) = \theta_j + \beta_i - \|z_j - w_i\|$$

Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.



**Value**

lsirm1pl\_fixed\_gamma\_mcar returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.

**References**

Little, R. J., & Rubin, D. B. (2019). *Statistical analysis with missing data* (Vol. 793). John Wiley & Sons.

**Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2), ncol=10, nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99
```

```
lsirm_result <- lsirm1pl_fixed_gamma_mcar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = TRUE,
missing_data = "mcar", missing.val = 99))
```

---

lsirm1pl\_mar

*1PL LSIRM for missing at random data.*


---

### Description

[lsirm1pl\\_mar](#) is used to fit 1PL LSIRM in incomplete data assumed to be missing at random. [lsirm1pl\\_mar](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm1pl_mar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_gamma = 0.025,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
  pr_sd_gamma = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  missing.val = 99,
  verbose = FALSE
)
```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
------	---

ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. default value is FALSE.

## Details

lsirm1pl\_mar models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$\text{logit}(P(Y_{j,i} = 1|\theta_j, \beta_i, \gamma, z_j, w_i)) = \theta_j + \beta_i - \gamma||z_j - w_i||$$

Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

## Value

lsirm1pl\_mar returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.

<code>mcmc_inf</code>	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
<code>map_inf</code>	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
<code>beta_estimate</code>	Posterior estimates of the beta parameter.
<code>theta_estimate</code>	Posterior estimates of the theta parameter.
<code>sigma_theta_estimate</code>	Posterior estimates of the standard deviation of theta.
<code>gamma_estimate</code>	posterior estimates of gamma parameter.
<code>z_estimate</code>	Posterior estimates of the z parameter.
<code>w_estimate</code>	Posterior estimates of the w parameter.
<code>imp_estimate</code>	Probability of imputating a missing value with 1.
<code>beta</code>	Posterior samples of the beta parameter.
<code>theta</code>	Posterior samples of the theta parameter.
<code>gamma</code>	Posterior samples of the gamma parameter.
<code>theta_sd</code>	Posterior samples of the standard deviation of theta.
<code>z</code>	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
<code>w</code>	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
<code>imp</code>	Imputation for missing Values using posterior samples.
<code>accept_beta</code>	Acceptance ratio for the beta parameter.
<code>accept_theta</code>	Acceptance ratio for the theta parameter.
<code>accept_z</code>	Acceptance ratio for the z parameter.
<code>accept_w</code>	Acceptance ratio for the w parameter.
<code>accept_gamma</code>	Acceptance ratio for the gamma parameter.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

## Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_mar(data)
```

```
# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = FALSE,
missing_data = 'mar', missing.val = 99))
```

---

lsirm1pl_mar_ss	<i>1PL LSIRM with model selection approach for missing at random data.</i>
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---

## Description

[lsirm1pl\\_mar\\_ss](#) is used to fit 1PL LSIRM with model selection approach based on spike-and-slab priors in incomplete data assumed to be missing at random. [lsirm1pl\\_mar\\_ss](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm1pl_mar_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_spike_mean = -3,
  pr_spike_sd = 1,
  pr_slab_mean = 0.5,
  pr_slab_sd = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  pr_xi_a = 1,
  pr_xi_b = 1,
  missing.val = 99,
  verbose = FALSE
)
```

**Arguments**

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

**Details**

lsirm1pl\_mar\_ss models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$\text{logit}(P(Y_{j,i} = 1 | \theta_j, \beta_i, \gamma, z_j, w_i)) = \theta_j + \beta_i - \gamma ||z_j - w_i||$$

Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References. lsirm1pl\_mar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

### Value

lsirm1pl\_mar\_ss returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
imp	Imputation for missing Values using posterior samples.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
pi_estimate	Posterior estimation of phi. inclusion probability of gamma. if estimation of phi is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space model with gamma > 0.
imp_estimate	Probability of imputating a missing value with 1.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons. Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: frequentist and Bayesian strategies. *The Annals of Statistics*, 33(2), 730-773.

## Examples

```
# generate example item response matrix
data      <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat  <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_mar_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = TRUE, fixed_gamma = FALSE,
missing_data = 'mar', missing = 99))
```

---

 lsirm1pl\_mcar

---

*1PL LSIRM for missing completely at random data.*


---

## Description

[lsirm1pl\\_mcar](#) is used to fit 1PL LSIRM in incomplete data assumed to be missing completely at random. [lsirm1pl\\_mcar](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm1pl_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_gamma = 0.025,
  jump_z = 0.5,
  jump_w = 0.5,
```



```

pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_mean_gamma = 0.5,
pr_sd_gamma = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
missing.val = 99,
verbose = FALSE
)

```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
missing.val	Numeric; A number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## Details

lsirm1pl\_mcar models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$\text{logit}(P(Y_{j,i} = 1|\theta_j, \beta_i, \gamma, z_j, w_i)) = \theta_j + \beta_i - \gamma||z_j - w_i||$$

Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.

## Value

lsirm1pl\_mcar returns an object of list containing the following components:

data	A data frame or matrix containing the variables used in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	Posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
gamma	Posterior samples of the gamma parameter.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

## Examples

```
# generate example item response matrix
data      <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat  <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_mcar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = FALSE,
missing_data = 'mcar', missing.val = 99))
```

---

lsirm1pl_mcar_ss	<i>1PL LSIRM with model selection approach for missing completely at random data.</i>
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---

## Description

[lsirm1pl\\_mcar\\_ss](#) is used to fit 1PL LSIRM with model selection approach based on spike-and-slab priors in incomplete data assumed to be missing completely at random. [lsirm1pl\\_mcar\\_ss](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm1pl_mcar_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_gamma = 1,
  jump_z = 0.5,
```

```

jump_w = 0.5,
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_spike_mean = -3,
pr_spike_sd = 1,
pr_slab_mean = 0.5,
pr_slab_sd = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_xi_a = 1,
pr_xi_b = 1,
missing.val = 99,
verbose = FALSE
)

```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.

pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm1pl\_mcar\_ss models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$\text{logit}(P(Y_{j,i} = 1 | \theta_j, \beta_i, \gamma, z_j, w_i)) = \theta_j + \beta_i - \gamma ||z_j - w_i||$$

Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References. lsirm1pl\_mcar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

### Value

lsirm1pl\_mcar\_ss returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.

theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.  
 Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: frequentist and Bayesian strategies. *The Annals of Statistics*, 33(2), 730-773.

## Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_mcar_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = TRUE, fixed_gamma = FALSE,
missing_data = 'mcar', missing.val = 99))
```

---

```
lsirm1pl_normal_fixed_gamma
```

*1PL LSIRM fixing gamma to 1 with normal likelihood*

---

## Description

[lsirm1pl\\_normal\\_fixed\\_gamma](#) is used to fit 1PL LSIRM for continuous variable with gamma fixed to 1. [lsirm1pl\\_normal\\_fixed\\_gamma](#) factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

**Usage**

```
lsirm1pl_normal_fixed_gamma(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  pr_a_eps = 0.001,
  pr_b_eps = 0.001,
  verbose = FALSE
)
```

**Arguments**

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>ndim</code>	Integer; the dimension of the latent space. Default is 2.
<code>niter</code>	Integer; the total number of MCMC iterations to run. Default is 15000.
<code>nburn</code>	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
<code>nthin</code>	Integer; the number of MCMC iterations to thin. Default is 5.
<code>nprint</code>	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
<code>jump_beta</code>	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
<code>jump_theta</code>	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
<code>jump_z</code>	Numeric; the jumping rule for the z proposal density. Default is 0.5.
<code>jump_w</code>	Numeric; the jumping rule for the w proposal density. Default is 0.5.
<code>pr_mean_beta</code>	Numeric; the mean of the normal prior for beta. Default is 0.
<code>pr_sd_beta</code>	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
<code>pr_mean_theta</code>	Numeric; the mean of the normal prior for theta. Default is 0.
<code>pr_a_theta</code>	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.

pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. default value is FALSE.

### Details

lsirm1pl\_normal\_fixed\_gamma models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space:

$$Y_{j,i} = \theta_j + \beta_i - \|z_j - w_i\| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ .

### Value

lsirm1pl\_normal\_fixed\_gamma returns an object of list containing the following components:

data	A data frame or matrix containing the variables used in the model.
bic	A numeric value representing the Bayesian Information Criterion (BIC).
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.



accept\_z            Acceptance ratio for the z parameter.  
 accept\_w            Acceptance ratio for the w parameter.  
 sigma\_estimate    Posterior estimates of the standard deviation.  
 sigma                Posterior samples of the standard deviation.

### Examples

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_normal_fixed_gamma(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = TRUE))
```

---

```
lsirm1pl_normal_fixed_gamma_mar
IPL LSIRM fixing gamma to 1 with normal likelihood for missing at
random data.
```

---

### Description

[lsirm1pl\\_normal\\_fixed\\_gamma\\_mar](#) is used to fit IPL LSIRM for continuous variable with gamma fixed to 1 in incomplete data assumed to be missing at random.

[lsirm1pl\\_normal\\_fixed\\_gamma\\_mar](#) factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm1pl_normal_fixed_gamma_mar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
```

```

jump_z = 0.5,
jump_w = 0.5,
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_a_eps = 0.001,
pr_b_eps = 0.001,
missing.val = 99,
verbose = FALSE
)

```

### Arguments

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>ndim</code>	Integer; the dimension of the latent space. Default is 2.
<code>niter</code>	Integer; the total number of MCMC iterations to run. Default is 15000.
<code>nburn</code>	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
<code>nthin</code>	Integer; the number of MCMC iterations to thin. Default is 5.
<code>nprint</code>	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
<code>jump_beta</code>	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
<code>jump_theta</code>	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
<code>jump_z</code>	Numeric; the jumping rule for the z proposal density. Default is 0.5.
<code>jump_w</code>	Numeric; the jumping rule for the w proposal density. Default is 0.5.
<code>pr_mean_beta</code>	Numeric; the mean of the normal prior for beta. Default is 0.
<code>pr_sd_beta</code>	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
<code>pr_mean_theta</code>	Numeric; the mean of the normal prior for theta. Default is 0.
<code>pr_a_theta</code>	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
<code>pr_b_theta</code>	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
<code>pr_a_eps</code>	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
<code>pr_b_eps</code>	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
<code>missing.val</code>	Numeric; a number to replace missing values. Default is 99.
<code>verbose</code>	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

**Details**

lsirm1pl\_normal\_fixed\_gamma\_mar models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space:

$$Y_{j,i} = \theta_j + \beta_i - \|z_j - w_i\| + e_{ji}$$

where the error  $e_{ji} \sim N(0, \sigma^2)$ . Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

**Value**

lsirm1pl\_normal\_fixed\_gamma\_mar returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
imp_estimate	Probability of imputating a missing value with 1.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
imp	Imputation for missing Values using posterior samples.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
sigma_estimate	Posterior estimates of the standard deviation.
sigma	Posterior samples of the standard deviation.

**Examples**

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_normal_fixed_gamma_mar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = TRUE,
missing_data = "mar", missing.val = 99))
```

---

```
lsirm1pl_normal_fixed_gamma_mcar
```

*1PL LSIRM fixing gamma to 1 with normal likelihood for missing completely at random data.*

---

**Description**

[lsirm1pl\\_normal\\_fixed\\_gamma\\_mcar](#) is used to fit 1PL LSIRM for continuous variable with gamma fixed to 1 in incomplete data assumed to be missing completely at random.

[lsirm1pl\\_normal\\_fixed\\_gamma\\_mcar](#) factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

**Usage**

```
lsirm1pl_normal_fixed_gamma_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_a_theta = 0.001,
```

```

    pr_b_theta = 0.001,
    pr_a_eps = 0.001,
    pr_b_eps = 0.001,
    missing.val = 99,
    verbose = FALSE
  )

```

## Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## Details

lsirm1pl\_normal\_fixed\_gamma\_mcar models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space:

$$Y_{j,i} = \theta_j + \beta_i - \|z_j - w_i\| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ . Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.

### Value

`lsirm1pl_normal_fixed_gamma_mcar` returns an object of list containing the following components:

<code>data</code>	Data frame or matrix containing the variables in the model.
<code>missing.val</code>	A number to replace missing values.
<code>bic</code>	Numeric value with the corresponding BIC.
<code>mcmc_inf</code>	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
<code>map_inf</code>	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
<code>beta_estimate</code>	Posterior estimates of the beta parameter.
<code>theta_estimate</code>	Posterior estimates of the theta parameter.
<code>sigma_theta_estimate</code>	Posterior estimates of the standard deviation of theta.
<code>z_estimate</code>	Posterior estimates of the z parameter.
<code>w_estimate</code>	Posterior estimates of the w parameter.
<code>beta</code>	Posterior samples of the beta parameter.
<code>theta</code>	Posterior samples of the theta parameter.
<code>theta_sd</code>	Posterior samples of the standard deviation of theta.
<code>z</code>	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
<code>w</code>	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
<code>accept_beta</code>	Acceptance ratio for the beta parameter.
<code>accept_theta</code>	Acceptance ratio for the theta parameter.
<code>accept_z</code>	Acceptance ratio for the z parameter.
<code>accept_w</code>	Acceptance ratio for the w parameter.
<code>sigma_estimate</code>	Posterior estimates of the standard deviation.
<code>sigma</code>	Posterior samples of the standard deviation.

### Examples

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1), ncol=10, nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2), ncol=10, nrow=50)
```

```
# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_normal_fixed_gamma_mcar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = TRUE,
missing_data = "mcar", missing.val = 99))
```

---

lsirm1pl\_normal\_mar     *IPL LSIRM with normal likelihood for missing at random data.*

---

### Description

`lsirm1pl_normal_mar` is used to fit LSIRM for continuous variable with 1pl in incomplete data assumed to be missing at random. `lsirm1pl_normal_mar` factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm1pl_normal_mar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
  pr_sd_gamma = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  pr_a_eps = 0.001,
  pr_b_eps = 0.001,
  missing.val = 99,
  verbose = FALSE
)
```

**Arguments**

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

**Details**

lsirm1pl\_normal\_mar models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_{j,i} = \theta_j + \beta_i - \gamma \|z_j - w_i\| + e_{ji}$$

where the error  $e_{ji} \sim N(0, \sigma^2)$  Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.



**Value**

lsirm1pl\_normal\_mar returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
imp_estimate	Probability of imputating a missing value with 1.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
imp	Imputation for missing Values using posterior samples.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
sigma_estimate	Posterior estimates of the standard deviation.
sigma	Posterior samples of the standard deviation.

**References**

Little, R. J., & Rubin, D. B. (2019). *Statistical analysis with missing data* (Vol. 793). John Wiley & Sons.

**Examples**

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_normal_mar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = FALSE,
missing_data = 'mar', missing.val = 99))
```

---

```
lsirm1pl_normal_mar_ss
```

*1PL LSIRM with normal likelihood and model selection approach for missing at random data.*

---

**Description**

`lsirm1pl_normal_mar_ss` is used to fit 1PL LSIRM with model selection approach based on spike-and-slab priors for continuous variable with 1pl in incomplete data assumed to be missing at random. `lsirm1pl_normal_mar_ss` factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

**Usage**

```
lsirm1pl_normal_mar_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
```

```

pr_spike_mean = -3,
pr_spike_sd = 1,
pr_slab_mean = 0.5,
pr_slab_sd = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_a_eps = 0.001,
pr_b_eps = 0.001,
pr_xi_a = 0.001,
pr_xi_b = 0.001,
missing.val = 99,
verbose = FALSE
)

```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.

pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm1pl\_normal\_mar\_ss models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ . Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References. lsirm1pl\_normal\_mar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

### Value

lsirm1pl\_normal\_mar\_ss returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.

theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
imp	Imputation for missing Values using posterior samples.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
pi_estimate	Posterior estimation of phi. inclusion probability of gamma. if estimation of phi is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space model with gamma > 0.
imp_estimate	Probability of imputating a missing value with 1.
sigma_estimate	Posterior estimates of the standard deviation.
sigma	Posterior samples of the standard deviation.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons. Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: frequentist and Bayesian strategies. *The Annals of Statistics*, 33(2), 730-773.

## Examples

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_normal_mar_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = TRUE, fixed_gamma = FALSE,
missing_data = 'mar', missing = 99))
```

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lsirm1pl\_normal\_mcar *1PL LSIRM with normal likelihood for missing completely at random data.*

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### Description

`lsirm1pl_normal_mcar` is used to fit LSIRM with 1pl in incomplete data assumed to be missing completely at random. `lsirm1pl_normal_mcar` factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm1pl_normal_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
  pr_sd_gamma = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  pr_a_eps = 0.001,
  pr_b_eps = 0.001,
  missing.val = 99,
  verbose = FALSE
)
```

### Arguments

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>ndim</code>	Integer; the dimension of the latent space. Default is 2.
<code>niter</code>	Integer; the total number of MCMC iterations to run. Default is 15000.

nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## Details

lsirm1pl\_normal\_mcar models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$  Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing at random assumption and data augmentation, see References.

**Value**

`lsirm1pl_normal_mcar` returns an object of list containing the following components:

<code>data</code>	A data frame or matrix containing the variables used in the model.
<code>missing.val</code>	A number to replace missing values.
<code>bic</code>	Numeric value with the corresponding BIC.
<code>mcmc_inf</code>	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
<code>map_inf</code>	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
<code>beta_estimate</code>	Posterior estimates of the beta parameter.
<code>theta_estimate</code>	Posterior estimates of the theta parameter.
<code>sigma_theta_estimate</code>	Posterior estimates of the standard deviation of theta.
<code>gamma_estimate</code>	Posterior estimates of gamma parameter.
<code>z_estimate</code>	Posterior estimates of the z parameter.
<code>w_estimate</code>	Posterior estimates of the w parameter.
<code>beta</code>	Posterior samples of the beta parameter.
<code>theta</code>	Posterior samples of the theta parameter.
<code>theta_sd</code>	Posterior samples of the standard deviation of theta.
<code>gamma</code>	Posterior samples of the gamma parameter.
<code>z</code>	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
<code>w</code>	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
<code>accept_beta</code>	Acceptance ratio for the beta parameter.
<code>accept_theta</code>	Acceptance ratio for the theta parameter.
<code>accept_z</code>	Acceptance ratio for the z parameter.
<code>accept_w</code>	Acceptance ratio for the w parameter.
<code>accept_gamma</code>	Acceptance ratio for the gamma parameter.
<code>sigma_estimate</code>	Posterior estimates of the standard deviation.
<code>sigma</code>	Posterior samples of the standard deviation.

**References**

Little, R. J., & Rubin, D. B. (2019). *Statistical analysis with missing data* (Vol. 793). John Wiley & Sons.



**Examples**

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1), ncol=10, nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2), ncol=10, nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_normal_mcar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = FALSE,
missing_data = 'mcar', missing.val = 99))
```

---

```
lsirm1pl_normal_mcar_ss
```

*1PL LSIRM with normal likelihood and model selection approach for missing completely at random data.*

---

**Description**

`lsirm1pl_normal_mcar_ss` is used to fit LSIRM with model selection approach based on spike-and-slab priors for continuous variable with 1pl in incomplete data assumed to be missing completely at random. `lsirm1pl_normal_mcar_ss` factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. The resulting latent space provides an interaction map that represents interactions between respondents and items.

**Usage**

```
lsirm1pl_normal_mcar_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
```

```

pr_spike_mean = -3,
pr_spike_sd = 1,
pr_slab_mean = 0.5,
pr_slab_sd = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_a_eps = 0.001,
pr_b_eps = 0.001,
pr_xi_a = 0.001,
pr_xi_b = 0.001,
missing.val = 99,
verbose = FALSE
)

```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.

pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm1pl\_normal\_mcar\_ss models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_{j,i} = \theta_j + \beta_i - \gamma \|z_j - w_i\| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ . Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing at random assumption and data augmentation, see References. lsirm1pl\_normal\_mcar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

### Value

lsirm1pl\_normal\_mcar\_ss returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.

theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
sigma_estimate	Posterior estimates of the standard deviation.
sigma	Posterior samples of the standard deviation.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons. Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: frequentist and Bayesian strategies. *The Annals of Statistics*, 33(2), 730-773.

## Examples

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm1pl_normal_mcar_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = TRUE, fixed_gamma = FALSE,
missing_data = 'mcar', missing.val = 99))
```

---

lsirm1pl\_normal\_o      *1PL LSIRM with normal likelihood.*

---

### Description

`lsirm1pl_normal_o` is used to fit LSIRM for continuous variable with 1pl. `lsirm1pl_normal_o` factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm1pl_normal_o(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
  pr_sd_gamma = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  pr_a_eps = 0.001,
  pr_b_eps = 0.001,
  verbose = FALSE
)
```

### Arguments

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>ndim</code>	Integer; the dimension of the latent space. Default is 2.
<code>niter</code>	Integer; the total number of MCMC iterations to run. Default is 15000.
<code>nburn</code>	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
<code>nthin</code>	Integer; the number of MCMC iterations to thin. Default is 5.

nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm1pl\_normal\_o models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ .

### Value

lsirm1pl\_normal\_o returns an object of list containing the following components:

data	Data frame or matrix containing the variables used in the model.
bic	A numeric value representing the Bayesian Information Criterion (BIC).
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.

theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	Posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
gamma	Posterior samples of the gamma parameter.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
sigma_estimate	Posterior estimates of the standard deviation.
sigma	Posterior samples of the standard deviation.

### Examples

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1), ncol=10, nrow=50)

lsirm_result <- lsirm1pl_normal_o(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = FALSE))
```

---

lsirm1pl\_normal\_ss      *1PL LSIRM with normal likelihood and model selection approach.*

---

### Description

`lsirm1pl_normal_ss` is used to fit LSIRM with model selection approach based on spike-and-slab priors for continuous variable with 1pl. LSIRM factorizes continuous item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. The resulting latent space provides an interaction map that represents interactions between respondents and items.

**Usage**

```
lsirm1pl_normal_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_spike_mean = -3,
  pr_spike_sd = 1,
  pr_slab_mean = 0.5,
  pr_slab_sd = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  pr_a_eps = 0.001,
  pr_b_eps = 0.001,
  pr_xi_a = 0.001,
  pr_xi_b = 0.001,
  verbose = FALSE
)
```

**Arguments**

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>ndim</code>	Integer; the dimension of the latent space. Default is 2.
<code>niter</code>	Integer; the total number of MCMC iterations to run. Default is 15000.
<code>nburn</code>	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
<code>nthin</code>	Integer; the number of MCMC iterations to thin. Default is 5.
<code>nprint</code>	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
<code>jump_beta</code>	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
<code>jump_theta</code>	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
<code>jump_gamma</code>	Numeric; the jumping rule for the gamma proposal density. Default is 1.0.
<code>jump_z</code>	Numeric; the jumping rule for the z proposal density. Default is 0.5.



jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; mean of spike prior for log gamma default value is -3.
pr_spike_sd	Numeric; standard deviation of spike prior for log gamma default value is 1.
pr_slab_mean	Numeric; mean of spike prior for log gamma default value is 0.5.
pr_slab_sd	Numeric; standard deviation of spike prior for log gamma default value is 1.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_xi_a	Numeric; first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; second shape parameter of beta prior for latent variable xi. Default is 1.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm1pl\_normal\_ss models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$Y_{j,i} = \theta_j + \beta_i - \gamma||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ . lsirm1pl\_normal\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

### Value

lsirm1pl\_normal\_ss returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
bic	Numeric value with the corresponding BIC.
mcmc_inf	number of mcmc iteration, burn-in periods, and thinning intervals.
map_inf	value of log maximum a posterior and iteration number which have log maximum a posterior.
beta_estimate	posterior estimation of beta.
theta_estimate	posterior estimation of theta.

<code>sigma_theta_estimate</code>	posterior estimation of standard deviation of theta.
<code>sigma_estimate</code>	posterior estimation of standard deviation.
<code>gamma_estimate</code>	posterior estimation of gamma.
<code>z_estimate</code>	posterior estimation of z.
<code>w_estimate</code>	posterior estimation of w.
<code>pi_estimate</code>	posterior estimation of phi. inclusion probability of gamma. if estimation of phi is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space model with gamma > 0.
<code>beta</code>	posterior samples of beta.
<code>theta</code>	posterior samples of theta.
<code>theta_sd</code>	posterior samples of standard deviation of theta.
<code>sigma</code>	posterior samples of standard deviation.
<code>gamma</code>	posterior samples of gamma.
<code>z</code>	posterior samples of z. The output is 3-dimensional matrix with last axis represent the dimension of latent space.
<code>w</code>	posterior samples of w. The output is 3-dimensional matrix with last axis represent the dimension of latent space.
<code>pi</code>	posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
<code>accept_beta</code>	accept ratio of beta.
<code>accept_theta</code>	accept ratio of theta.
<code>accept_w</code>	accept ratio of w.
<code>accept_z</code>	accept ratio of z.
<code>accept_gamma</code>	accept ratio of gamma.

## References

Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies (Vol. 33). *The Annals of Statistics*

## Examples

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1), ncol=10, nrow=50)

lsirm_result <- lsirm1pl_normal_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = TRUE, fixed_gamma = FALSE))
```

---

lsirm1pl_o	<i>1PL LSIRM.</i>
------------	-------------------

---

### Description

`lsirm1pl_o` is used to fit 1PL LSIRM. `lsirm1pl_o` factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm1pl_o(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_gamma = 0.025,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
  pr_sd_gamma = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  verbose = FALSE
)
```

### Arguments

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>ndim</code>	Integer; the dimension of the latent space. Default is 2.
<code>niter</code>	Integer; the total number of MCMC iterations to run. Default is 15000.
<code>nburn</code>	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
<code>nthin</code>	Integer; the number of MCMC iterations to thin. Default is 5.
<code>nprint</code>	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.

jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm1pl\_o models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$\text{logit}(P(Y_{j,i} = 1|\theta_j, \beta_i, \gamma, z_j, w_i)) = \theta_j + \beta_i - \gamma||z_j - w_i||$$

### Value

lsirm1pl\_o returns an object of list containing the following components:

data	Data frame or matrix containing the variables used in the model.
bic	A numeric value representing the Bayesian Information Criterion (BIC).
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	Posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.

theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
gamma	Posterior samples of the gamma parameter.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.

### Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

lsirm_result <- lsirm1pl_o(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = FALSE, fixed_gamma = FALSE))
```

---

lsirm1pl\_ss

*1PL LSIRM with model selection approach.*

---

### Description

[lsirm1pl\\_ss](#) is used to fit 1PL LSIRM with model selection approach based on spike-and-slab priors. LSIRM factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm1pl_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
```

```

jump_gamma = 1,
jump_z = 0.5,
jump_w = 0.5,
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_spike_mean = -3,
pr_spike_sd = 1,
pr_slab_mean = 0.5,
pr_slab_sd = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_xi_a = 1,
pr_xi_b = 1,
verbose = FALSE
)

```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.

pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm1pl\_ss models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term:

$$\text{logit}(P(Y_{j,i} = 1|\theta_j, \beta_i, \gamma, z_j, w_i)) = \theta_j + \beta_i - \gamma||z_j - w_i||$$

lsirm1pl\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

### Value

lsirm1pl\_ss returns an object of list containing the following components:

data	Data frame or matrix containing the variables used in the model.
bic	A numeric value representing the Bayesian Information Criterion (BIC).
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	Posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
gamma	Posterior samples of the gamma parameter.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.

accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
pi_estimate	Posterior estimation of phi. inclusion probability of gamma. if estimation of phi is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space model with gamma > 0.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.

## References

Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies (Vol. 33). The Annals of Statistics

## Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

lsirm_result <- lsirm1pl_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm1pl(spikenslab = TRUE, fixed_gamma = FALSE))
```

---

lsirm2pl

*Fit a 2pl LSIRM for binary and continuous item response data*

---

## Description

`lsirm2pl` integrates all functions related to 2PL LSIRM. Various 2PL LSIRM function can be used by setting the `spikenslab`, `fixed_gamma`, and `missing_data` arguments.

This function can be used regardless of the data type, providing a unified approach to model fitting.

## Usage

```
lsirm2pl(
  data,
  spikenslab = FALSE,
  fixed_gamma = FALSE,
  missing_data = NA,
  chains = 1,
  multicore = 1,
  seed = NA,
```



```

    ndim = 2,
    niter = 15000,
    nburn = 2500,
    nthin = 5,
    nprint = 500,
    jump_beta = 0.4,
    jump_theta = 1,
    jump_alpha = 1,
    jump_z = 0.5,
    jump_w = 0.5,
    pr_mean_beta = 0,
    pr_sd_beta = 1,
    pr_mean_theta = 0,
    pr_a_theta = 0.001,
    pr_b_theta = 0.001,
    pr_mean_alpha = 0.5,
    pr_sd_alpha = 1,
    ...
)

```

### Arguments

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>spikenslab</code>	Logical; specifies whether to use a model selection approach. Default is FALSE.
<code>fixed_gamma</code>	Logical; indicates whether to fix gamma at 1. Default is FALSE.
<code>missing_data</code>	Character; the type of missing data assumed. Options are NA, "mar", or "mcar". Default is NA.
<code>chains</code>	Integer; the number of MCMC chains to run. Default is 1.
<code>multicore</code>	Integer; the number of cores to use for parallel execution. Default is 1.
<code>seed</code>	Integer; the seed number for MCMC fitting. Default is NA.
<code>ndim</code>	Integer; the dimension of the latent space. Default is 2.
<code>niter</code>	Integer; the total number of MCMC iterations to run. Default is 15000.
<code>nburn</code>	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
<code>nthin</code>	Integer; the number of MCMC iterations to thin. Default is 5.
<code>nprint</code>	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
<code>jump_beta</code>	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
<code>jump_theta</code>	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
<code>jump_alpha</code>	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
<code>jump_z</code>	Numeric; the jumping rule for the z proposal density. Default is 0.5.
<code>jump_w</code>	Numeric; the jumping rule for the w proposal density. Default is 0.5.

pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
...	Additional arguments for the for various settings. Refer to the functions in the Details.

## Details

Additional arguments and return values for each function are documented in the respective function's description.

\* For 2PL LSIRM with data included missing value are detailed in [lsirm2pl\\_mar](#) and [lsirm2pl\\_mcar](#).

\* For 2PL LSIRM using the spike-and-slab model selection approach are detailed in [lsirm2pl\\_ss](#).

\* For continuous version of 2PL LSIRM are detailed in [lsirm2pl\\_normal\\_o](#).

For 2PL LSIRM with binary item response data, the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$\text{logit}(P(Y_{j,i} = 1 | \theta_j, \alpha_i, \beta_i, \gamma, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - \gamma ||z_j - w_i||$$

For 2PL LSIRM with continuous item response data, the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$

## Value

lsirm2pl returns an object of list. The basic return list containing the following components:

data	A data frame or matrix containing the variables used in the model.
bic	A numeric value representing the Bayesian Information Criterion (BIC).
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.

map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
alpha_estimate	posterior estimates of alpha parameter..
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
alpha	Posterior samples of the alpha parameter.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_alpha	Acceptance ratio for the alpha parameter.
...	Additional return values for various settings. Refer to the functions in the Details.

**Note**

If both `spikenslab` and `fixed_gamma` are set TRUE, it returns error because both are related to `gamma`.

**See Also**

The 2PL LSIRM for binary item response data as following:

`lsirm2pl_o`, `lsirm2pl_fixed_gamma`, `lsirm2pl_mar`, `lsirm2pl_mcar`, `lsirm2pl_fixed_gamma_mar`, `lsirm2pl_fixed_gamma_mcar`, `lsirm2pl_ss`, `lsirm2pl_mar_ss`, and `lsirm2pl_mcar_ss`

The 2PL LSIRM for continuous item response data as following:

`lsirm2pl_normal_o`, `lsirm2pl_normal_fixed_gamma`, `lsirm2pl_normal_mar`, `lsirm2pl_normal_mcar`, `lsirm1pl_norm`, `lsirm2pl_normal_fixed_gamma_mcar`, `lsirm2pl_normal_ss`, `lsirm2pl_normal_mar_ss`, `lsirm2pl_normal_mcar_ss`

**Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)
lsirm_result <- lsirm2pl(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data~lsirm2pl())
```

---

lsirm2pl\_fixed\_gamma    *2PL LSIRM fixing gamma to 1.*

---

**Description**

[lsirm2pl\\_fixed\\_gamma](#) is used to fit 2PL LSIRM fixing gamma to 1. [lsirm2pl\\_fixed\\_gamma](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

**Usage**

```
lsirm2pl_fixed_gamma(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  verbose = FALSE
)
```

**Arguments**

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

**Details**

lsirm2pl\_fixed\_gamma models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$\text{logit}(P(Y_{j,i} = 1|\theta_j, \alpha_i, \beta_i, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - ||z_j - w_i||$$

**Value**

lsirm2pl\_fixed\_gamma returns an object of list containing the following components:

lsirm1pl\_fixed\_gamma returns an object of list containing the following components:

<code>data</code>	Data frame or matrix containing the variables in the model.
<code>bic</code>	Numeric value with the corresponding BIC.
<code>mcmc_inf</code>	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
<code>map_inf</code>	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
<code>beta_estimate</code>	Posterior estimates of the beta parameter.
<code>theta_estimate</code>	Posterior estimates of the theta parameter.
<code>sigma_theta_estimate</code>	Posterior estimates of the standard deviation of theta.
<code>z_estimate</code>	Posterior estimates of the z parameter.
<code>w_estimate</code>	Posterior estimates of the w parameter.
<code>beta</code>	Posterior samples of the beta parameter.
<code>theta</code>	Posterior samples of the theta parameter.
<code>theta_sd</code>	Posterior samples of the standard deviation of theta.
<code>z</code>	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
<code>w</code>	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
<code>accept_beta</code>	Acceptance ratio for the beta parameter.
<code>accept_theta</code>	Acceptance ratio for the theta parameter.
<code>accept_z</code>	Acceptance ratio for the z parameter.
<code>accept_w</code>	Acceptance ratio for the w parameter.
<code>alpha_estimate</code>	Posterior estimates of the alpha parameter.
<code>alpha</code>	Posterior estimates of the alpha parameter.
<code>accept_alpha</code>	Acceptance ratio for the alpha parameter.

### Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

lsirm_result <- lsirm2pl_fixed_gamma(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = TRUE))
```

---

 lsirm2pl\_fixed\_gamma\_mar

*2PL LSIRM fixing gamma to 1 for missing at random data.*


---

## Description

[lsirm2pl\\_fixed\\_gamma\\_mar](#) is used to fit 2PL LSIRM fixing gamma to 1 in incomplete data assumed to be missing at random. [lsirm2pl\\_fixed\\_gamma\\_mar](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. Unlike 1pl model, 2pl model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm2pl_fixed_gamma_mar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  missing.val = 99,
  verbose = FALSE
)
```

## Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.

nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
missing.val	Numeric; A number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm2pl\_fixed\_gamma\_mar models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$\text{logit}(P(Y_{j,i} = 1|\theta_j, \alpha_i, \beta_i, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - ||z_j - w_i||$$

Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

### Value

lsirm2pl\_fixed\_gamma\_mar returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.



mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
imp_estimate	Probability of imputating a missing value with 1.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
imp	Imputation for missing Values using posterior samples.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

## Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_fixed_gamma_mar(data)
```

```
# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = TRUE,
                                     missing_data = "mar"))
```

---

```
lsirm2pl_fixed_gamma_mcar
```

*2PL LSIRM fixing gamma to 1 for missing completely at random data.*

---

## Description

[lsirm2pl\\_fixed\\_gamma\\_mcar](#) is used to fit 2PL LSIRM fixing gamma to 1 in incomplete data assumed to be missing completely at random. [lsirm2pl\\_fixed\\_gamma\\_mcar](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. Unlike 1pl model, 2pl model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm2pl_fixed_gamma_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  missing.val = 99,
  verbose = FALSE
)
```

**Arguments**

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
missing.val	Numeric; A number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

**Details**

lsirm2pl\_fixed\_gamma\_mcar models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$\text{logit}(P(Y_{j,i} = 1|\theta_j, \alpha_i, \beta_i, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - ||z_j - w_i||$$

Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.

**Value**

`lsirm2pl_fixed_gamma_mar` returns an object of list containing the following components:

<code>data</code>	Data frame or matrix containing the variables in the model.
<code>missing.val</code>	A number to replace missing values.
<code>bic</code>	Numeric value with the corresponding BIC.
<code>mcmc_inf</code>	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
<code>map_inf</code>	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
<code>beta_estimate</code>	Posterior estimates of the beta parameter.
<code>theta_estimate</code>	Posterior estimates of the theta parameter.
<code>sigma_theta_estimate</code>	Posterior estimates of the standard deviation of theta.
<code>z_estimate</code>	Posterior estimates of the z parameter.
<code>w_estimate</code>	Posterior estimates of the w parameter.
<code>beta</code>	Posterior samples of the beta parameter.
<code>theta</code>	Posterior samples of the theta parameter.
<code>theta_sd</code>	Posterior samples of the standard deviation of theta.
<code>z</code>	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
<code>w</code>	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
<code>accept_beta</code>	Acceptance ratio for the beta parameter.
<code>accept_theta</code>	Acceptance ratio for the theta parameter.
<code>accept_z</code>	Acceptance ratio for the z parameter.
<code>accept_w</code>	Acceptance ratio for the w parameter.
<code>alpha_estimate</code>	Posterior estimates of the alpha parameter.
<code>alpha</code>	Posterior estimates of the alpha parameter.
<code>accept_alpha</code>	Acceptance ratio for the alpha parameter.

**References**

Little, R. J., & Rubin, D. B. (2019). *Statistical analysis with missing data* (Vol. 793). John Wiley & Sons.

**Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2), ncol=10, nrow=50)
```

```
# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_fixed_gamma_mcar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = TRUE,
  missing_data = "mcar"))
```

---

lsirm2pl\_mar

---

*2PL LSIRM for missing at random data.*


---

### Description

[lsirm2pl\\_mar](#) is used to fit 2PL LSIRM in incomplete data assumed to be missing at random. [lsirm2pl\\_mar](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect in a latent space, while considering the missing element under the assumption of missing at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_mar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_gamma = 0.025,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
  pr_sd_gamma = 1,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  missing.val = 99,
```

```

    verbose = FALSE
  )

```

### Arguments

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>ndim</code>	Integer; the dimension of the latent space. Default is 2.
<code>niter</code>	Integer; the total number of MCMC iterations to run. Default is 15000.
<code>nburn</code>	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
<code>nthin</code>	Integer; the number of MCMC iterations to thin. Default is 5.
<code>nprint</code>	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
<code>jump_beta</code>	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
<code>jump_theta</code>	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
<code>jump_alpha</code>	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
<code>jump_gamma</code>	Numeric; the jumping rule for the gamma proposal density. Default is 0.025.
<code>jump_z</code>	Numeric; the jumping rule for the z proposal density. Default is 0.5.
<code>jump_w</code>	Numeric; the jumping rule for the w proposal density. Default is 0.5.
<code>pr_mean_beta</code>	Numeric; the mean of the normal prior for beta. Default is 0.
<code>pr_sd_beta</code>	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
<code>pr_mean_theta</code>	Numeric; the mean of the normal prior for theta. Default is 0.
<code>pr_mean_gamma</code>	Numeric; mean of log normal prior for gamma. Default is 0.5.
<code>pr_sd_gamma</code>	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
<code>pr_mean_alpha</code>	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
<code>pr_sd_alpha</code>	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
<code>pr_a_theta</code>	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
<code>pr_b_theta</code>	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
<code>missing.val</code>	Numeric; A number to replace missing values. Default is 99.
<code>verbose</code>	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## Details

lsirm2pl\_mar models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$\text{logit}(P(Y_{j,i} = 1|\theta_j, \alpha_i, \beta_i, \gamma, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - \gamma||z_j - w_i||$$

Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

## Value

lsirm2pl\_mar returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
imp_estimate	Probability of imputating a missing value with 1.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
imp	Imputation for missing Values using posterior samples.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.

accept\_w            Acceptance ratio for the w parameter.  
 accept\_gamma      Acceptance ratio for the gamma parameter.  
 alpha\_estimate    Posterior estimates of the alpha parameter.  
 alpha              Posterior estimates of the alpha parameter.  
 accept\_alpha      Acceptance ratio for the alpha parameter.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

## Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2), ncol=10, nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_mar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = FALSE,
  missing_data = "mar"))
```

---

lsirm2pl_mar_ss	<i>2PL LSIRM with model selection approach for missing at random data.</i>
-----------------	--

---

## Description

`lsirm2pl_mar_ss` is used to fit 2PL LSIRM based on spike-and-slab priors in incomplete data assumed to be missing at random. `lsirm2pl_mar_ss` factorizes item response matrix into column-wise item effect, row-wise respondent effect in a latent space, while considering the missing element under the assumption of missing at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.



**Usage**

```
lsirm2pl_mar_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_spike_mean = -3,
  pr_spike_sd = 1,
  pr_slab_mean = 0.5,
  pr_slab_sd = 1,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  pr_xi_a = 1,
  pr_xi_b = 1,
  missing.val = 99,
  verbose = FALSE
)
```

**Arguments**

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>ndim</code>	Integer; the dimension of the latent space. Default is 2.
<code>niter</code>	Integer; the total number of MCMC iterations to run. Default is 15000.
<code>nburn</code>	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
<code>nthin</code>	Integer; the number of MCMC iterations to thin. Default is 5.
<code>nprint</code>	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
<code>jump_beta</code>	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
<code>jump_theta</code>	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
<code>jump_alpha</code>	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.

jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## Details

lsirm2pl\_mar\_ss models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$\text{logit}(P(Y_{j,i} = 1 | \theta_j, \alpha_i, \beta_i, \gamma, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - \gamma ||z_j - w_i||$$

Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References. lsirm2pl\_mar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

## Value

lsirm2pl\_mar\_ss returns an object of list containing the following components:

data                    Data frame or matrix containing the variables in the model.

missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
imp	Imputation for missing Values using posterior samples.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
pi_estimate	Posterior estimation of phi. inclusion probability of gamma. if estimation of phi is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space model with gamma > 0.
imp_estimate	Probability of imputating a missing value with 1.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

## References

- Little, R. J., & Rubin, D. B. (2019). *Statistical analysis with missing data* (Vol. 793). John Wiley & Sons.
- Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies. *The Annals of Statistics*, 33(2), 730-773.

## Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2), ncol=10, nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_mar_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = TRUE, fixed_gamma = FALSE,
  missing_data = "mar"))
```

---

 lsirm2pl\_mcar

*2PL LSIRM for missing completely at random data.*


---

## Description

[lsirm2pl\\_mcar](#) is used to fit 2PL LSIRM in incomplete data assumed to be missing completely at random. [lsirm2pl\\_mcar](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm2pl_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_gamma = 0.025,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
```

```

pr_mean_gamma = 0.5,
pr_sd_gamma = 1,
pr_mean_alpha = 0.5,
pr_sd_alpha = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
missing.val = 99,
verbose = FALSE
)

```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
missing.val	Numeric; A number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## Details

lsirm2pl\_mcar models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$\text{logit}(P(Y_{j,i} = 1|\theta_j, \alpha_i, \beta_i, \gamma, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - \gamma||z_j - w_i||$$

Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.

## Value

lsirm2pl\_mar returns an object of list containing the following components:

data	A data frame or matrix containing the variables used in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	Posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
gamma	Posterior samples of the gamma parameter.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

## Examples

```
# generate example item response matrix
data      <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

# generate example missing indicator matrix
missing_mat  <- matrix(rbinom(500, size = 1, prob = 0.2), ncol=10, nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_mcar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = FALSE,
                                     missing_data = "mcar"))
```

---

lsirm2pl_mcar_ss	<i>2PL LSIRM with model selection approach for missing completely at random data.</i>
------------------	---

---

## Description

[lsirm2pl\\_mar\\_ss](#) is used to fit 2PL LSIRM based on spike-and-slab priors in incomplete data assumed to be missing completely at random. [lsirm2pl\\_mar\\_ss](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm2pl_mcar_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
```

```

jump_gamma = 1,
jump_z = 0.5,
jump_w = 0.5,
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_spike_mean = -3,
pr_spike_sd = 1,
pr_slab_mean = 0.5,
pr_slab_sd = 1,
pr_mean_alpha = 0.5,
pr_sd_alpha = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_xi_a = 1,
pr_xi_b = 1,
missing.val = 99,
verbose = FALSE
)

```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.



pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm2pl\_mcar\_ss models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$\text{logit}(P(Y_{j,i} = 1 | \theta_j, \alpha_i, \beta_i, \gamma, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - \gamma ||z_j - w_i||$$

Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References. lsirm2pl\_mcar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

### Value

lsirm2pl\_mar\_ss returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.

gamma_estimate	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons. Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies. *The Annals of Statistics*, 33(2), 730-773.

## Examples

```
# generate example item response matrix
data      <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

# generate example missing indicator matrix
missing_mat  <- matrix(rbinom(500, size = 1, prob = 0.2), ncol=10, nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_mcar_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = TRUE, fixed_gamma = FALSE,
  missing_data = "mcar"))
```

---

 lsirm2pl\_normal\_fixed\_gamma

*2PL LSIRM fixing gamma to 1 with normal likelihood*


---

## Description

[lsirm2pl\\_normal\\_fixed\\_gamma](#) is used to fit 2PL LSIRM with gamma fixed to 1 for continuous variable. [lsirm2pl\\_normal\\_fixed\\_gamma](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm2pl_normal_fixed_gamma(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  pr_a_eps = 0.001,
  pr_b_eps = 0.001,
  verbose = FALSE
)
```

## Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.

nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

`lsirm2pl_normal_fixed_gamma` models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$

### Value

`lsirm2pl_normal_fixed_gamma` returns an object of list containing the following components:

data	A data frame or matrix containing the variables used in the model.
bic	A numeric value representing the Bayesian Information Criterion (BIC).

mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
sigma_estimate	Posterior estimates of the standard deviation.
sigma	Posterior samples of the standard deviation.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

### Examples

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1), ncol=10, nrow=50)

lsirm_result <- lsirm2pl_normal_fixed_gamma(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = TRUE))
```

---

```
lsirm2pl_normal_fixed_gamma_mar
```

*2PL LSIRM fixing gamma to 1 with normal likelihood for missing at random data.*

---

### Description

[lsirm2pl\\_normal\\_fixed\\_gamma\\_mar](#) is used to fit 2PL LSIRM with gamma fixed to 1 for continuous variable in incomplete data assumed to be missing at random.

[lsirm2pl\\_normal\\_fixed\\_gamma\\_mar](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_normal_fixed_gamma_mar(  
  data,  
  ndim = 2,  
  niter = 15000,  
  nburn = 2500,  
  nthin = 5,  
  nprint = 500,  
  jump_beta = 0.4,  
  jump_theta = 1,  
  jump_alpha = 1,  
  jump_z = 0.5,  
  jump_w = 0.5,  
  pr_mean_beta = 0,  
  pr_sd_beta = 1,  
  pr_mean_theta = 0,  
  pr_mean_alpha = 0.5,  
  pr_sd_alpha = 1,  
  pr_a_theta = 0.001,  
  pr_b_theta = 0.001,  
  pr_a_eps = 0.001,  
  pr_b_eps = 0.001,  
  missing.val = 99,  
  verbose = FALSE  
)
```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding
------	---

	item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## Details

lsirm2pl\_normal\_fixed\_gamma\_mar models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$  Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

**Value**

`lsirm2pl_normal_fixed_gamma_mar` returns an object of list containing the following components:

<code>data</code>	Data frame or matrix containing the variables in the model.
<code>missing.val</code>	A number to replace missing values.
<code>bic</code>	Numeric value with the corresponding BIC.
<code>mcmc_inf</code>	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
<code>map_inf</code>	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
<code>beta_estimate</code>	Posterior estimates of the beta parameter.
<code>theta_estimate</code>	Posterior estimates of the theta parameter.
<code>sigma_theta_estimate</code>	Posterior estimates of the standard deviation of theta.
<code>z_estimate</code>	Posterior estimates of the z parameter.
<code>w_estimate</code>	Posterior estimates of the w parameter.
<code>imp_estimate</code>	Probability of imputating a missing value with 1.
<code>beta</code>	Posterior samples of the beta parameter.
<code>theta</code>	Posterior samples of the theta parameter.
<code>theta_sd</code>	Posterior samples of the standard deviation of theta.
<code>z</code>	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
<code>w</code>	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
<code>imp</code>	Imputation for missing Values using posterior samples.
<code>accept_beta</code>	Acceptance ratio for the beta parameter.
<code>accept_theta</code>	Acceptance ratio for the theta parameter.
<code>accept_z</code>	Acceptance ratio for the z parameter.
<code>accept_w</code>	Acceptance ratio for the w parameter.
<code>sigma_estimate</code>	Posterior estimates of the standard deviation.
<code>sigma</code>	Posterior samples of the standard deviation.
<code>alpha_estimate</code>	Posterior estimates of the alpha parameter.
<code>alpha</code>	Posterior estimates of the alpha parameter.
<code>accept_alpha</code>	Acceptance ratio for the alpha parameter.



**Examples**

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_normal_fixed_gamma_mar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = TRUE,
  missing_data = "mar"))
```

---

```
lsirm2pl_normal_fixed_gamma_mcar
```

*2PL LSIRM fixing gamma to 1 with normal likelihood for missing completely at random data.*

---

**Description**

[lsirm2pl\\_normal\\_fixed\\_gamma\\_mcar](#) is used to fit 2PL LSIRM with gamma fixed to 1 for continuous variable in incomplete data assumed to be missing completely at random.

[lsirm2pl\\_normal\\_fixed\\_gamma\\_mcar](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

**Usage**

```
lsirm2pl_normal_fixed_gamma_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_z = 0.5,
  jump_w = 0.5,
```

```

pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_mean_alpha = 0.5,
pr_sd_alpha = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_a_eps = 0.001,
pr_b_eps = 0.001,
missing.val = 99,
verbose = FALSE
)

```

### Arguments

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>ndim</code>	Integer; the dimension of the latent space. Default is 2.
<code>niter</code>	Integer; the total number of MCMC iterations to run. Default is 15000.
<code>nburn</code>	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
<code>nthin</code>	Integer; the number of MCMC iterations to thin. Default is 5.
<code>nprint</code>	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
<code>jump_beta</code>	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
<code>jump_theta</code>	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
<code>jump_alpha</code>	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
<code>jump_z</code>	Numeric; the jumping rule for the z proposal density. Default is 0.5.
<code>jump_w</code>	Numeric; the jumping rule for the w proposal density. Default is 0.5.
<code>pr_mean_beta</code>	Numeric; the mean of the normal prior for beta. Default is 0.
<code>pr_sd_beta</code>	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
<code>pr_mean_theta</code>	Numeric; the mean of the normal prior for theta. Default is 0.
<code>pr_mean_alpha</code>	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
<code>pr_sd_alpha</code>	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
<code>pr_a_theta</code>	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
<code>pr_b_theta</code>	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
<code>pr_a_eps</code>	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.

pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm2pl\_normal\_fixed\_gamma\_mcar models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$  Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.

### Value

lsirm2pl\_normal\_fixed\_gamma\_mcar returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.

accept\_theta    Acceptance ratio for the theta parameter.  
 accept\_z        Acceptance ratio for the z parameter.  
 accept\_w        Acceptance ratio for the w parameter.  
 sigma\_estimate   Posterior estimates of the standard deviation.  
 sigma            Posterior samples of the standard deviation.  
 alpha\_estimate   Posterior estimates of the alpha parameter.  
 alpha            Posterior estimates of the alpha parameter.  
 accept\_alpha    Acceptance ratio for the alpha parameter.

### Examples

```

# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1), ncol=10, nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2), ncol=10, nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_normal_fixed_gamma_mcar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = TRUE,
  missing_data = "mcar"))

```

---

lsirm2pl\_normal\_mar    *2PL LSIRM with normal likelihood and missing at random data.*

---

### Description

[lsirm2pl\\_normal\\_mar](#) is used to fit 2PL LSIRM for continuous variable in incomplete data assumed to be missing at random. [lsirm2pl\\_normal\\_mar](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```

lsirm2pl_normal_mar(
  data,
  ndim = 2,
  niter = 15000,

```

```

nburn = 2500,
nthin = 5,
nprint = 500,
jump_beta = 0.4,
jump_theta = 1,
jump_alpha = 1,
jump_gamma = 1,
jump_z = 0.5,
jump_w = 0.5,
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_mean_gamma = 0.5,
pr_sd_gamma = 1,
pr_mean_alpha = 0.5,
pr_sd_alpha = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_a_eps = 0.001,
pr_b_eps = 0.001,
missing.val = 99,
verbose = FALSE
)

```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.

pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm2pl\_normal\_mar models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$  Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References.

### Value

lsirm2pl\_normal\_mar returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.

gamma_estimate	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
imp_estimate	Probability of imputating a missing value with 1.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
imp	Imputation for missing Values using posterior samples.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
sigma_estimate	Posterior estimates of the standard deviation.
sigma	Posterior samples of the standard deviation.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

## Examples

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_normal_mar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = FALSE,
missing_data = "mar"))
```

---

 lsirm2pl\_normal\_mar\_ss

*2pl LSIRM with normal likelihood and model selection approach for missing at random data.*

---

### Description

`lsirm2pl_normal_mar_ss` is used to fit 2pl LSIRM with model selection approach based on spike-and-slab priors for continuous variable in incomplete data assumed to be missing at random. `lsirm2pl_normal_mar_ss` factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while considering the missing element under the assumption of missing at random. Unlike 1pl model, 2pl model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_normal_mar_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_spike_mean = -3,
  pr_spike_sd = 1,
  pr_slab_mean = 0.5,
  pr_slab_sd = 1,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_eps = 0.001,
  pr_b_eps = 0.001,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  pr_xi_a = 0.001,
  pr_xi_b = 0.001,
  missing.val = 99,
```



```

    verbose = FALSE
  )

```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.

pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm2pl\_normal\_mar\_ss models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$  Under the assumption of missing at random, the model takes the missing element into consideration in the sampling procedure. For the details of missing at random assumption and data augmentation, see References. lsirm2pl\_normal\_mcar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

### Value

lsirm2pl\_normal\_mar\_ss returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.

w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
imp	Imputation for missing Values using posterior samples.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
pi_estimate	Posterior estimation of phi. inclusion probability of gamma. if estimation of phi is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space model with gamma > 0.
imp_estimate	Probability of imputating a missing value with 1.
sigma_estimate	Posterior estimates of the standard deviation.
sigma	Posterior samples of the standard deviation.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons. Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies. *The Annals of Statistics*, 33(2), 730-773.

## Examples

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_normal_mar_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = TRUE, fixed_gamma = FALSE,
missing_data = "mar"))
```

---

lsirm2pl_normal_mcar	<i>2PL LSIRM with normal likelihood and missing completely at random data.</i>
----------------------	--

---

### Description

[lsirm2pl\\_normal\\_mcar](#) is used to fit 2PL LSIRM for continuous variable in incomplete data assumed to be missing completely at random. [lsirm2pl\\_normal\\_mcar](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_normal_mcar(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
  pr_sd_gamma = 1,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  pr_a_eps = 0.001,
  pr_b_eps = 0.001,
  missing.val = 99,
  verbose = FALSE
)
```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding
------	---

	item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## Details

lsirm2pl\_normal\_mcar models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ . Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References.

### Value

lsirm2pl\_normal\_mcar returns an object of list containing the following components:

data	A data frame or matrix containing the variables used in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	Posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
gamma	Posterior samples of the gamma parameter.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
sigma_estimate	Posterior estimates of the standard deviation.
sigma	Posterior samples of the standard deviation.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

## References

Little, R. J., & Rubin, D. B. (2019). Statistical analysis with missing data (Vol. 793). John Wiley & Sons.

## Examples

```
# generate example (continuous) item response matrix
data      <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat  <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_normal_mcar(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = FALSE,
                                     missing_data = "mcar"))
```

---

```
lsirm2pl_normal_mcar_ss
```

*2PL LSIRM with normal likelihood and model selection approach for missing completely at random data.*

---

## Description

[lsirm2pl\\_normal\\_mcar\\_ss](#) is used to fit 2PL LSIRM with model selection approach based on spike-and-slab priors for continuous variable in incomplete data assumed to be missing completely at random. [lsirm2pl\\_normal\\_mcar\\_ss](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space, while ignoring the missing element under the assumption of missing completely at random. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

## Usage

```
lsirm2pl_normal_mcar_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
```

```

jump_theta = 1,
jump_alpha = 1,
jump_gamma = 1,
jump_z = 0.5,
jump_w = 0.5,
pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_spike_mean = -3,
pr_spike_sd = 1,
pr_slab_mean = 0.5,
pr_slab_sd = 1,
pr_mean_alpha = 0.5,
pr_sd_alpha = 1,
pr_a_eps = 0.001,
pr_b_eps = 0.001,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_xi_a = 0.001,
pr_xi_b = 0.001,
missing.val = 99,
verbose = FALSE
)

```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.



pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_eps	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
missing.val	Numeric; a number to replace missing values. Default is 99.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## Details

lsirm2pl\_normal\_mcar\_ss models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ . Under the assumption of missing completely at random, the model ignores the missing element in doing inference. For the details of missing completely at random assumption and data augmentation, see References. lsirm2pl\_normal\_mcar\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

## Value

lsirm2pl\_normal\_mcar\_ss returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
missing.val	A number to replace missing values.
bic	Numeric value with the corresponding BIC.

mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
gamma	Posterior samples of the gamma parameter.
theta_sd	Posterior samples of the standard deviation of theta.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
sigma_estimate	Posterior estimates of the standard deviation.
sigma	Posterior samples of the standard deviation.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

## References

- Little, R. J., & Rubin, D. B. (2019). *Statistical analysis with missing data* (Vol. 793). John Wiley & Sons.
- Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies. *The Annals of Statistics*, 33(2), 730-773.

**Examples**

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1),ncol=10,nrow=50)

# generate example missing indicator matrix
missing_mat <- matrix(rbinom(500, size = 1, prob = 0.2),ncol=10,nrow=50)

# make missing value with missing indicator matrix
data[missing_mat==1] <- 99

lsirm_result <- lsirm2pl_normal_mcar_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = TRUE, fixed_gamma = FALSE,
missing_data = "mcar"))
```

---

lsirm2pl\_normal\_o      *2PL LSIRM with normal likelihood*

---

**Description**

`lsirm2pl_normal_o` is used to fit 2PL LSIRM for continuous variable. `lsirm2pl_normal_o` factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

**Usage**

```
lsirm2pl_normal_o(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_gamma = 0.025,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
```

```

pr_sd_gamma = 1,
pr_mean_alpha = 0.5,
pr_sd_alpha = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_a_eps = 0.001,
pr_b_eps = 0.001,
verbose = FALSE
)

```

### Arguments

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>ndim</code>	Integer; the dimension of the latent space. Default is 2.
<code>niter</code>	Integer; the total number of MCMC iterations to run. Default is 15000.
<code>nburn</code>	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
<code>nthin</code>	Integer; the number of MCMC iterations to thin. Default is 5.
<code>nprint</code>	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
<code>jump_beta</code>	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
<code>jump_theta</code>	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
<code>jump_alpha</code>	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
<code>jump_gamma</code>	Numeric; the jumping rule for the gamma proposal density. Default is 0.025
<code>jump_z</code>	Numeric; the jumping rule for the z proposal density. Default is 0.5.
<code>jump_w</code>	Numeric; the jumping rule for the w proposal density. Default is 0.5.
<code>pr_mean_beta</code>	Numeric; the mean of the normal prior for beta. Default is 0.
<code>pr_sd_beta</code>	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
<code>pr_mean_theta</code>	Numeric; the mean of the normal prior for theta. Default is 0.
<code>pr_mean_gamma</code>	Numeric; mean of log normal prior for gamma. Default is 0.5.
<code>pr_sd_gamma</code>	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
<code>pr_mean_alpha</code>	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
<code>pr_sd_alpha</code>	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
<code>pr_a_theta</code>	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
<code>pr_b_theta</code>	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
<code>pr_a_eps</code>	Numeric; the shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.

pr_b_eps	Numeric; the scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm2pl\_normal\_o models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$

### Value

lsirm2pl\_normal\_o returns an object of list containing the following components:

data	Data frame or matrix containing the variables used in the model.
bic	A numeric value representing the Bayesian Information Criterion (BIC).
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	Posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
gamma	Posterior samples of the gamma parameter.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.

accept\_w            Acceptance ratio for the w parameter.  
 accept\_gamma        Acceptance ratio for the gamma parameter.  
 sigma\_estimate     Posterior estimates of the standard deviation.  
 sigma                Posterior samples of the standard deviation.  
 alpha\_estimate     Posterior estimates of the alpha parameter.  
 alpha                Posterior estimates of the alpha parameter.  
 accept\_alpha        Acceptance ratio for the alpha parameter.

### Examples

```

# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1), ncol=10, nrow=50)
lsirm_result <- lsirm2pl_normal_o(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = FALSE))

```

---

lsirm2pl\_normal\_ss        *2PL LSIRM with normal likelihood and model selection approach.*

---

### Description

[lsirm2pl\\_normal\\_ss](#) is used to fit 2PL LSIRM for continuous variable with model selection approach. [lsirm2pl\\_normal\\_ss](#) factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```

lsirm2pl_normal_ss(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_gamma = 1,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,

```

```

pr_sd_beta = 1,
pr_mean_theta = 0,
pr_spike_mean = -3,
pr_spike_sd = 1,
pr_slab_mean = 0.5,
pr_slab_sd = 1,
pr_mean_alpha = 0.5,
pr_sd_alpha = 1,
pr_a_eps = 0.001,
pr_b_eps = 0.001,
pr_a_theta = 0.001,
pr_b_theta = 0.001,
pr_xi_a = 0.001,
pr_xi_b = 0.001,
verbose = FALSE
)

```

### Arguments

data	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; mean of spike prior for log gamma default value is -3.
pr_spike_sd	Numeric; standard deviation of spike prior for log gamma default value is 1.
pr_slab_mean	Numeric; mean of spike prior for log gamma default value is 0.5.
pr_slab_sd	Numeric; standard deviation of spike prior for log gamma default value is 1.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.

pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_eps	Numeric; shape parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_b_eps	Numeric; scale parameter of inverse gamma prior for variance of data likelihood. Default is 0.001.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; second shape parameter of beta prior for latent variable xi. Default is 1.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

### Details

lsirm2pl\_normal\_ss models the continuous value of response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$Y_{j,i} = \theta_j + \beta_i - \gamma ||z_j - w_i|| + e_{j,i}$$

where the error  $e_{j,i} \sim N(0, \sigma^2)$ . lsirm2pl\_normal\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

### Value

lsirm2pl\_normal\_ss returns an object of list containing the following components:

data	Data frame or matrix containing the variables in the model.
bic	Numeric value with the corresponding BIC.
mcmc_inf	number of mcmc iteration, burn-in periods, and thinning intervals.
map_inf	value of log maximum a posterior and iteration number which have log maximum a posterior.
beta_estimate	posterior estimation of beta.
theta_estimate	posterior estimation of theta.
sigma_theta_estimate	posterior estimation of standard deviation of theta.
sigma_estimate	posterior estimation of standard deviation.
gamma_estimate	posterior estimation of gamma.
z_estimate	posterior estimation of z.
w_estimate	posterior estimation of w.



pi_estimate	posterior estimation of phi. inclusion probability of gamma. if estimation of phi is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space model with gamma > 0.
beta	posterior samples of beta.
theta	posterior samples of theta.
theta_sd	posterior samples of standard deviation of theta.
sigma	posterior samples of standard deviation.
gamma	posterior samples of gamma.
z	posterior samples of z. The output is 3-dimensional matrix with last axis represent the dimension of latent space.
w	posterior samples of w. The output is 3-dimensional matrix with last axis represent the dimension of latent space.
pi	posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
accept_beta	accept ratio of beta.
accept_theta	accept ratio of theta.
accept_w	accept ratio of w.
accept_z	accept ratio of z.
accept_gamma	accept ratio of gamma.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

## References

Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: frequentist and Bayesian strategies. *The Annals of Statistics*, 33(2), 730-773.

## Examples

```
# generate example (continuous) item response matrix
data <- matrix(rnorm(500, mean = 0, sd = 1), ncol=10, nrow=50)

lsirm_result <- lsirm2pl_normal_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = TRUE, fixed_gamma = FALSE))
```

lsirm2pl\_o

*2PL LSIRM.***Description**

`lsirm2pl_o` is used to fit 2PL LSIRM. `lsirm2pl_o` factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

**Usage**

```
lsirm2pl_o(
  data,
  ndim = 2,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  jump_alpha = 1,
  jump_gamma = 0.025,
  jump_z = 0.5,
  jump_w = 0.5,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_mean_gamma = 0.5,
  pr_sd_gamma = 1,
  pr_mean_alpha = 0.5,
  pr_sd_alpha = 1,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001,
  verbose = FALSE
)
```

**Arguments**

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
<code>ndim</code>	Integer; the dimension of the latent space. Default is 2.
<code>niter</code>	Integer; the total number of MCMC iterations to run. Default is 15000.

nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the gamma proposal density. Default is 0.025.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_mean_gamma	Numeric; mean of log normal prior for gamma. Default is 0.5.
pr_sd_gamma	Numeric; standard deviation of log normal prior for gamma. Default is 1.0.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## Details

lsirm2pl\_o models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$\text{logit}(P(Y_{j,i} = 1 | \theta_j, \alpha_i, \beta_i, \gamma, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - \gamma ||z_j - w_i||$$

## Value

lsirm2pl\_o returns an object of list containing the following components:

data	Data frame or matrix containing the variables used in the model.
bic	A numeric value representing the Bayesian Information Criterion (BIC).
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.

map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	Posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
gamma	Posterior samples of the gamma parameter.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

### Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

lsirm_result <- lsirm2pl_o(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = FALSE, fixed_gamma = FALSE))
```

---

`lsirm2pl_ss`*2PL LSIRM with model selection approach.*

---

### Description

`lsirm2pl_ss` is used to fit 2PL LSIRM with model selection approach based on spike-and-slab priors. `lsirm2pl_ss` factorizes item response matrix into column-wise item effect, row-wise respondent effect and further embeds interaction effect in a latent space. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect. The resulting latent space provides an interaction map that represents interactions between respondents and items.

### Usage

```
lsirm2pl_ss(  
  data,  
  ndim = 2,  
  niter = 15000,  
  nburn = 2500,  
  nthin = 5,  
  nprint = 500,  
  jump_beta = 0.4,  
  jump_theta = 1,  
  jump_alpha = 1,  
  jump_gamma = 1,  
  jump_z = 0.5,  
  jump_w = 0.5,  
  pr_mean_beta = 0,  
  pr_sd_beta = 1,  
  pr_mean_theta = 0,  
  pr_spike_mean = -3,  
  pr_spike_sd = 1,  
  pr_slab_mean = 0.5,  
  pr_slab_sd = 1,  
  pr_mean_alpha = 0.5,  
  pr_sd_alpha = 1,  
  pr_a_theta = 0.001,  
  pr_b_theta = 0.001,  
  pr_xi_a = 1,  
  pr_xi_b = 1,  
  verbose = FALSE  
)
```

### Arguments

<code>data</code>	Matrix; a binary or continuous item response matrix for analysis. Each row represents a respondent, and each column contains responses to the corresponding item.
-------------------	---

ndim	Integer; the dimension of the latent space. Default is 2.
niter	Integer; the total number of MCMC iterations to run. Default is 15000.
nburn	Integer; the number of initial MCMC iterations to discard as burn-in. Default is 2500.
nthin	Integer; the number of MCMC iterations to thin. Default is 5.
nprint	Integer; the interval at which MCMC samples are displayed during execution. Default is 500.
jump_beta	Numeric; the jumping rule for the beta proposal density. Default is 0.4.
jump_theta	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_alpha	Numeric; the jumping rule for the alpha proposal density. Default is 1.0.
jump_gamma	Numeric; the jumping rule for the theta proposal density. Default is 1.0.
jump_z	Numeric; the jumping rule for the z proposal density. Default is 0.5.
jump_w	Numeric; the jumping rule for the w proposal density. Default is 0.5.
pr_mean_beta	Numeric; the mean of the normal prior for beta. Default is 0.
pr_sd_beta	Numeric; the standard deviation of the normal prior for beta. Default is 1.0.
pr_mean_theta	Numeric; the mean of the normal prior for theta. Default is 0.
pr_spike_mean	Numeric; the mean of spike prior for log gamma. Default is -3.
pr_spike_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_slab_mean	Numeric; the mean of spike prior for log gamma. Default is 0.5.
pr_slab_sd	Numeric; the standard deviation of spike prior for log gamma. Default is 1.
pr_mean_alpha	Numeric; the mean of the log normal prior for alpha. Default is 0.5.
pr_sd_alpha	Numeric; the standard deviation of the log normal prior for alpha. Default is 1.0.
pr_a_theta	Numeric; the shape parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_b_theta	Numeric; the scale parameter of the inverse gamma prior for the variance of theta. Default is 0.001.
pr_xi_a	Numeric; the first shape parameter of beta prior for latent variable xi. Default is 1.
pr_xi_b	Numeric; the second shape parameter of beta prior for latent variable xi. Default is 1.
verbose	Logical; If TRUE, MCMC samples are printed for each nprint. Default is FALSE.

## Details

lsirm2pl\_ss models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$  and the distance between latent position  $w_i$  of item  $i$  and latent position  $z_j$  of respondent  $j$  in the shared metric space, with  $\gamma$  represents the weight of the distance term. For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$\text{logit}(P(Y_{j,i} = 1|\theta_j, \alpha_i, \beta_i, z_j, w_i)) = \theta_j * \alpha_i + \beta_i - \gamma||z_j - w_i||$$

lsirm2pl\_ss model include model selection approach based on spike-and-slab priors for log gamma. For detail of spike-and-slab priors, see References.

**Value**

lsirm2pl\_ss returns an object of list containing the following components:

data	Data frame or matrix containing the variables used in the model.
bic	A numeric value representing the Bayesian Information Criterion (BIC).
mcmc_inf	Details about the number of MCMC iterations, burn-in periods, and thinning intervals.
map_inf	The log maximum a posteriori (MAP) value and the iteration number at which this MAP value occurs.
beta_estimate	Posterior estimates of the beta parameter.
theta_estimate	Posterior estimates of the theta parameter.
sigma_theta_estimate	Posterior estimates of the standard deviation of theta.
gamma_estimate	Posterior estimates of gamma parameter.
z_estimate	Posterior estimates of the z parameter.
w_estimate	Posterior estimates of the w parameter.
beta	Posterior samples of the beta parameter.
theta	Posterior samples of the theta parameter.
theta_sd	Posterior samples of the standard deviation of theta.
gamma	Posterior samples of the gamma parameter.
z	Posterior samples of the z parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
w	Posterior samples of the w parameter, represented as a 3-dimensional matrix where the last axis denotes the dimension of the latent space.
accept_beta	Acceptance ratio for the beta parameter.
accept_theta	Acceptance ratio for the theta parameter.
accept_z	Acceptance ratio for the z parameter.
accept_w	Acceptance ratio for the w parameter.
accept_gamma	Acceptance ratio for the gamma parameter.
pi_estimate	Posterior estimation of phi. inclusion probability of gamma. if estimation of phi is less than 0.5, choose Rasch model with gamma = 0, otherwise latent space model with gamma > 0.
pi	Posterior samples of phi which is indicator of spike and slab prior. If phi is 1, log gamma follows the slab prior, otherwise follows the spike prior.
alpha_estimate	Posterior estimates of the alpha parameter.
alpha	Posterior estimates of the alpha parameter.
accept_alpha	Acceptance ratio for the alpha parameter.

**References**

Ishwaran, H., & Rao, J. S. (2005). Spike and slab variable selection: Frequentist and Bayesian strategies. *The Annals of Statistics*, 33(2), 730-773.

**Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

lsirm_result <- lsirm2pl_ss(data)

# The code following can achieve the same result.
lsirm_result <- lsirm(data ~ lsirm2pl(spikenslab = TRUE, fixed_gamma = FALSE))
```

---

onepl

*1PL Rasch model.*


---

**Description**

`onepl` is used to fit 1PL Rasch model.

**Usage**

```
onepl(
  data,
  niter = 15000,
  nburn = 2500,
  nthin = 5,
  nprint = 500,
  jump_beta = 0.4,
  jump_theta = 1,
  pr_mean_beta = 0,
  pr_sd_beta = 1,
  pr_mean_theta = 0,
  pr_a_theta = 0.001,
  pr_b_theta = 0.001
)
```

**Arguments**

<code>data</code>	Matrix; binary item response matrix to be analyzed. Each row is assumed to be respondent and its column values are assumed to be response to the corresponding item.
<code>niter</code>	Numeric; number of iterations to run MCMC sampling. default value is 15000.
<code>nburn</code>	Numeric; number of initial, pre-thinning, MCMC iterations to discard. default value is 2500.
<code>nthin</code>	Numeric; number of thinning, MCMC iterations to discard. default value is 5.
<code>nprint</code>	Numeric; MCMC samples is displayed during execution of MCMC chain for each nprint. default value is 500.
<code>jump_beta</code>	Numeric; jumping rule of the proposal density for beta. default value is 0.4.



jump_theta	Numeric; jumping rule of the proposal density for theta. default value is 1.0.
pr_mean_beta	Numeric; mean of normal prior for beta. default value is 0.
pr_sd_beta	Numeric; standard deviation of normal prior for beta. default value is 1.0.
pr_mean_theta	Numeric; mean of normal prior for theta. default value is 0.
pr_a_theta	Numeric; shape parameter of inverse gamma prior for variance of theta. default value is 0.001.
pr_b_theta	Numeric; scale parameter of inverse gamma prior for variance of theta. default value is 0.001.

### Details

onepl models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$ :

$$\text{logit}(P(Y_{j,i} = 1|\theta_j, \beta_i)) = \theta_j + \beta_i$$

### Value

onepl returns an object of list containing the following components:

beta_estimate	posterior estimation of beta.
theta_estimate	posterior estimation of theta.
sigma_theta_estimate	posterior estimation of standard deviation of theta.
beta	posterior samples of beta.
theta	posterior samples of theta.
theta_sd	posterior samples of standard deviation of theta.
accept_beta	accept ratio of beta.
accept_theta	accept ratio of theta.

### Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

result <- onepl(data)
```

---

plot	<i>Plotting the interaction map or summarizing the parameter estimate of fitted LSIRM with box plot.</i>
------	--

---

### Description

`plot` is used to plot the interaction map of fitted LSIRM or summarizing the parameter estimate of fitted LSIRM with box plot.

### Usage

```
plot(
  object,
  ...,
  option = "interaction",
  rotation = FALSE,
  cluster = NA,
  which.clust = "item",
  interact = FALSE,
  chain.idx = 1
)
```

### Arguments

object	Object of class <code>lsirm</code> .
...	Additional arguments for the corresponding function.
option	Character; If value is "interaction", draw the interaction map that represents interactions between respondents and items. If value is "beta", draw the boxplot for the posterior samples of beta. If value is "theta", draw the distribution of the theta estimates per total test score for the data. If value is "alpha", draw the boxplot for the posterior samples of alpha. The "alpha" is only available for 2PL LSIRM.
rotation	Logical; If TRUE the latent positions are visualized after oblique (oblimin) rotation.
cluster	Character; If value is "neyman" the cluster result are visualized by Point Process Cluster Analysis. If value is "spectral", spectral clustering method applied. Default is NA.
which.clust	Character; Choose which values to clustering. "resp" is the option for respondent and "item" is the option for items. Default is "item".
interact	Logical; If TRUE, draw the interaction map interactively.
chain.idx	Numeric; Index of MCMC chain. Default is 1.

### Value

`plot` returns the interaction map or boxplot for parameter estimate.

## Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)
lsirm_result <- lsirm(data ~ lsirm1pl())
plot(lsirm_result)

# use oblique rotation
plot(lsirm_result, rotation = TRUE)

# interaction map interactively
plot(lsirm_result, interact = TRUE)

# clustering the respondents or items
plot(lsirm_result, cluster = TRUE)
```

---

```
print.summary.lsirm Print the summary the result of LSIRM
```

---

## Description

[print.summary.lsirm](#) is used to print summary the result of LSIRM.

## Usage

```
## S3 method for class 'summary.lsirm'
print(x, ...)
```

## Arguments

x	List; summary of LSIRM with <code>summary.lsirm</code> .
...	Additional arguments.

## Value

`print.summary.lsirm` return a summary of LSIRM.

## Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)
lsirm_result <- lsirm(data ~ lsirm1pl())
summary(lsirm_result)
```

---

summary.lsirm                      *Summary the result of LSIRM*

---

### Description

`summary` is used to summary the result of LSIRM.

### Usage

```
## S3 method for class 'lsirm'
summary(object, chain.idx = 1, estimate = "mean", CI = 0.95, ...)
```

### Arguments

<code>object</code>	Object of class <code>lsirm</code> .
<code>chain.idx</code>	Numeric; Index of MCMC chain. Default is 1.
<code>estimate</code>	Character; Specifies the type of posterior estimate to provide for beta parameters. Options are "mean", "median", or "mode". Default is "mean".
<code>CI</code>	Numeric; The significance level for the highest posterior density interval (HPD) for the beta parameters. Default is 0.95.
<code>...</code>	Additional arguments.

### Value

`summary.lsirm` contains following elements. A print method is available.

<code>call</code>	R call used to fit the model.
<code>coef</code>	Covariate coefficients posterior means.
<code>mcmc.opt</code>	The number of mcmc iteration, burn-in periods, and thinning intervals.
<code>map.inf</code>	Value of log maximum a posterior and iteration number which have log maximum a posterior.
<code>BIC</code>	Numeric value with the corresponding Bayesian information criterion (BIC).
<code>method</code>	Which model is fitted.
<code>missing</code>	The assumed missing type. One of NA, "mar" and "mcar".
<code>dtype</code>	Type of input data (Binary or Continuous).
<code>ss</code>	Whether a model selection approach using the spike-slab prior is applied.

### Examples

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

# 1PL LSIRM object
lsirm_result <- lsirm(data ~ lsirm1pl())
summary(lsirm_result)
```

---

TDRI

*Inductive Reasoning Developmental Test*

---

### Description

TDRI dataset is the answer to Inductive Reasoning Developmental Test of 1,803 Brazilians with age varying from 5 to 85 years.

### Usage

```
data(TDRI)
```

### Format

A binary matrix with 1,803 rows and 56 columns.

### Details

It presents data from 1,803 Brazilians (52.5% female) with age varying from 5 to 85 years ( $M = 15.75$ ;  $SD = 12.21$ ) that answered to the Inductive Reasoning Developmental Test – IRDT, with 56 items designed to assess developmentally sequenced and hierarchically organized inductive reasoning.

### Source

[https://figshare.com/articles/dataset/TDRI\\_dataset\\_csv/3142321](https://figshare.com/articles/dataset/TDRI_dataset_csv/3142321)

---

twopl

*2PL Rasch model.*

---

### Description

`twopl` is used to fit 2PL Rasch model. Unlike 1PL model, 2PL model assumes the item effect can vary according to respondent, allowing additional parameter multiplied with respondent effect.

### Usage

```
twopl(  
  data,  
  niter = 15000,  
  nburn = 2500,  
  nthin = 5,  
  nprint = 500,  
  jump_beta = 0.4,  
  jump_theta = 1,  
  jump_alpha = 1,
```

```

pr_mean_beta = 0,
pr_sd_beta = 1,
pr_mean_theta = 0,
pr_mean_alpha = 0.5,
pr_sd_alpha = 1,
pr_a_theta = 0.001,
pr_b_theta = 0.001
)

```

### Arguments

data	Matrix; binary item response matrix to be analyzed. Each row is assumed to be respondent and its column values are assumed to be response to the corresponding item.
niter	Numeric; number of iterations to run MCMC sampling. default value is 15000.
nburn	Numeric; number of initial, pre-thinning, MCMC iterations to discard. default value is 2500.
nthin	Numeric; number of thinning, MCMC iterations to discard. default value is 5.
nprint	Numeric; MCMC samples is displayed during execution of MCMC chain for each nprint. default value is 500.
jump_beta	Numeric; jumping rule of the proposal density for beta. default value is 0.4.
jump_theta	Numeric; jumping rule of the proposal density for theta. default value is 1.0.
jump_alpha	Numeric; jumping rule of the proposal density for alpha default value is 1.0.
pr_mean_beta	Numeric; mean of normal prior for beta. default value is 0.
pr_sd_beta	Numeric; standard deviation of normal prior for beta. default value is 1.0.
pr_mean_theta	Numeric; mean of normal prior for theta. default value is 0.
pr_mean_alpha	Numeric; mean of normal prior for alpha. default value is 0.5.
pr_sd_alpha	Numeric; mean of normal prior for beta. default value is 1.0.
pr_a_theta	Numeric; shape parameter of inverse gamma prior for variance of theta. default value is 0.001.
pr_b_theta	Numeric; scale parameter of inverse gamma prior for variance of theta. default value is 0.001.

### Details

twopl models the probability of correct response by respondent  $j$  to item  $i$  with item effect  $\beta_i$ , respondent effect  $\theta_j$ . For 2pl model, the the item effect is assumed to have additional discrimination parameter  $\alpha_i$  multiplied by  $\theta_j$ :

$$\text{logit}(P(Y_{j,i} = 1 | \theta_j, \beta_i, \alpha_i)) = \theta_j * \alpha_i + \beta_i$$

**Value**

twopl returns an object of list containing the following components:

beta_estimate	posterior estimation of beta.
theta_estimate	posterior estimation of theta.
sigma_theta_estimate	posterior estimation of standard deviation of theta.
alpha_estimate	posterior estimation of alpha.
beta	posterior samples of beta.
theta	posterior samples of theta.
theta_sd	posterior samples of standard deviation of theta.
alpha	posterior samples of alpha.
accept_beta	accept ratio of beta.
accept_theta	accept ratio of theta.
accept_alpha	accept ratio of alpha.

**Examples**

```
# generate example item response matrix
data <- matrix(rbinom(500, size = 1, prob = 0.5), ncol=10, nrow=50)

result <- twopl(data)
```

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