Package 'GNE'

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Type Package

Title Computation of Generalized Nash Equilibria

Version 0.99-6

Description Compute standard and generalized Nash Equilibria of non-cooperative games. Optimization methods available are nonsmooth reformulation, fixed-point formulation, minimization problem and constrained-equation reformulation.
See e.g. Kanzow and Facchinei (2010), <doi:10.1007/s10479-009-0653-x>.

Depends R (>= 3.0.0), alabama, nleqslv, BB, SQUAREM, methods

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`bench.GNE` *Benchmark function*

Description

Benchmark function to compare GNE computational methods.

Usage

```
bench.GNE.nseq(xinit, ..., echo=FALSE, control=list())
bench.GNE.ceq(xinit, ..., echo=FALSE, control=list())
bench.GNE.fpeq(xinit, ..., echo=FALSE, control.outer=list(),
control.inner=list())
bench.GNE.minpb(xinit, ..., echo=FALSE, control.outer=list(),
control.inner=list())
```

Arguments

`xinit`	a numeric vector for the initial point.
`...`	further arguments to be passed to `GNE.nseq`, `GNE.ceq`, `GNE.fpeq` or `GNE.minpb`. NOT to the functions `func1` and `func2`.
`echo`	a logical to get some traces of the benchmark computation.
`control`, `control.outer`, `control.inner`	a list with control parameters to be passed to `GNE.xxx` function.

Details

Computing generalized Nash Equilibrium can be done in three different approaches.

(i) extended KKT system It consists in solving the non smooth extended Karush-Kuhn-Tucker (KKT) system $\Phi(z) = 0$. `func1` is Phi and `func2` is $JacPhi$.

(ii) fixed point approach It consists in solving equation $y(x) = x$. `func1` is y and `func2` is ?

(iii) gap function minimization It consists in minimizing a gap function $minV(x)$. `func1` is V and `func2` is $GradV$.

Value

For `GNE.bench.ceq` and `GNE.bench.nseq`, a `data.frame` is returned with columns:

`method` the name of the method.

`fctcall` the number of calls of the function.

`jaccall` the number of calls of the Jacobian.

`comptime` the computation time.

`normFx` the norm of the merit function at the final iterate.

`code` the exit code.

`localmethods` the name of the local method.

`globalmethods` the name of the globalization method.

`x` the final iterate.

For `GNE.bench.minpb`, a `data.frame` is returned with columns:

`method` the name of the method.

`minfncall.outer` the number of calls of the merit function.

`grminfncall.outer` the number of calls of the gradient of the merit function.

`gapfncall.inner` the number of calls of the gap function.

`grgapfncall.outer` the number of calls of the gradient of the gap function.

`comptime` the computation time.

`normFx` the norm of the merit function at the final iterate.

`code` the exit code.

`x` the final iterate.

For `GNE.bench.fpeq`, a `data.frame` is returned with columns:

`method` the name of the method.

`fpfncall.outer` the number of calls of the fixed-point function.

`merfncall.outer` the number of calls of the merit function.

`gapfncall.inner` the number of calls of the gap function.

`grgapfncall.outer` the number of calls of the gradient of the gap function.

`comptime` the computation time.

`normFx` the norm of the merit function at the final iterate.

`code` the exit code.

`x` the final iterate.

Author(s)

Christophe Dutang

References

F. Facchinei, A. Fischer & V. Piccialli (2009), *Generalized Nash equilibrium problems and Newton methods*, Math. Program.

A. von Heusinger (2009), *Numerical Methods for the Solution of the Generalized Nash Equilibrium Problem*, Ph. D. Thesis.

A. von Heusinger & J. Kanzow (2009), *Optimization reformulations of the generalized Nash equilibrium problem using Nikaido-Isoda-type functions*, Comput Optim Appl .

See Also

See `GNE.fpeq`, `GNE.minpb`, `GNE.ceq` and `GNE.nseq` for other approaches.

`CER` *Constrained Equation Reformulation*

Description

functions of the Constrained Equation Reformulation of the GNEP

Usage

```
funCER(z, dimx, dimlam,
grobj, arggrobj,
constr, argconstr,
grconstr, arggrconstr,
dimmu, joint, argjoint,
grjoint, arggrjoint,
echo=FALSE)

jacCER(z, dimx, dimlam,
heobj, argheobj,
constr, argconstr,
grconstr, arggrconstr,
heconstr, argheconstr,
dimmu, joint, argjoint,
grjoint, arggrjoint,
hejoint, arghejoint,
echo=FALSE)
```

Arguments

`z`	a numeric vector z containing x then lambda values.
`dimx`	dimension of x.
`dimlam`	dimension of lambda.

grobj	gradient of the objective function, see details.
arggrobj	a list of additional arguments of the objective gradient.
constr	constraint function, see details.
argconstr	a list of additional arguments of the constraint function.
grconstr	gradient of the constraint function, see details.
arggrconstr	a list of additional arguments of the constraint gradient.
dimmu	a vector of dimension for mu.
joint	joint function, see details.
argjoint	a list of additional arguments of the joint function.
grjoint	gradient of the joint function, see details.
arggrjoint	a list of additional arguments of the joint gradient.
heobj	Hessian of the objective function, see details.
argheobj	a list of additional arguments of the objective Hessian.
heconstr	Hessian of the constraint function, see details.
argheconstr	a list of additional arguments of the constraint Hessian.
hejoint	Hessian of the joint function, see details.
arghejoint	a list of additional arguments of the joint Hessian.
echo	a logical to show some traces.

Details

Compute the H function or the Jacobian of the H function defined in Dreves et al.(2009).

Arguments of the H function The arguments which are functions must respect the following features

grobj The gradient $GradObj$ of an objective function Obj (to be minimized) must have 3 arguments for $GradObj(z, playnum, ideriv)$: vector z, player number, derivative index , and optionnally additional arguments in arggrobj.

constr The constraint function g must have 2 arguments: vector z, player number, such that $g(z, playnum) <= 0$. Optionnally, g may have additional arguments in argconstr.

grconstr The gradient of the constraint function g must have 3 arguments: vector z, player number, derivative index, and optionnally additional arguments in arggrconstr.

Arguments of the Jacobian of H The arguments which are functions must respect the following features

heobj It must have 4 arguments: vector z, player number, two derivative indexes.

heconstr It must have 4 arguments: vector z, player number, two derivative indexes.

Optionnally, heobj and heconstr can have additional arguments argheobj and argheconstr.

See the example below.

Value

A vector for funCER or a matrix for jacCER.

Author(s)

Christophe Dutang

References

Dreves, A., Facchinei, F., Kanzow, C. and Sagratella, S. (2011), *On the solutions of the KKT conditions of generalized Nash equilibrium problems*, SIAM Journal on Optimization.

F. Facchinei, A. Fischer and V. Piccialli (2009), *Generalized Nash equilibrium problems and Newton methods*, Math. Program.

See Also

See also `GNE.ceq`.

Examples

```
#-------------------------------------------------------------------------------
# (1) Example 5 of von Facchinei et al. (2007)
#-------------------------------------------------------------------------------

dimx <- c(1, 1)
#Gr_x_j O_i(x)
grobj <- function(x, i, j)
{
if(i == 1)
res <- c(2*(x[1]-1), 0)
if(i == 2)
res <- c(0, 2*(x[2]-1/2))
res[j]
}
#Gr_x_k Gr_x_j O_i(x)
heobj <- function(x, i, j, k)
2 * (i == j && j == k)

dimlam <- c(1, 1)
#constraint function g_i(x)
g <- function(x, i)
sum(x[1:2]) - 1
#Gr_x_j g_i(x)
grg <- function(x, i, j)
1
#Gr_x_k Gr_x_j g_i(x)
heg <- function(x, i, j, k)
0


x0 <- rep(0, sum(dimx))
z0 <- c(x0, 2, 2, max(10, 5-g(x0, 1) ), max(10, 5-g(x0, 2) ) )
```

```
#true value is (3/4, 1/4, 1/2, 1/2)
funCER(z0, dimx, dimlam, grobj=grobj,
constr=g, grconstr=grg)

jacCER(z0, dimx, dimlam, heobj=heobj,
constr=g, grconstr=grg, heconstr=heg)



#------------------------------------------------------------------------------
# (2) Duopoly game of Krawczyk and Stanislav Uryasev (2000)
#------------------------------------------------------------------------------


#constants
myarg <- list(d= 20, lambda= 4, rho= 1)

dimx <- c(1, 1)
#Gr_x_j O_i(x)
grobj <- function(x, i, j, arg)
{
res <- -arg$rho * x[i]
if(i == j)
res <- res + arg$d - arg$lambda - arg$rho*(x[1]+x[2])
-res
}
#Gr_x_k Gr_x_j O_i(x)
heobj <- function(x, i, j, k, arg)
arg$rho * (i == j) + arg$rho * (j == k)


dimlam <- c(1, 1)
#constraint function g_i(x)
g <- function(x, i)
-x[i]
#Gr_x_j g_i(x)
grg <- function(x, i, j)
-1*(i == j)
#Gr_x_k Gr_x_j g_i(x)
heg <- function(x, i, j, k)
0

#true value is (16/3, 16/3, 0, 0)

x0 <- rep(0, sum(dimx))
z0 <- c(x0, 2, 2, max(10, 5-g(x0, 1) ), max(10, 5-g(x0, 2) ) )


funCER(z0, dimx, dimlam, grobj=grobj, arggrobj=myarg,
constr=g, grconstr=grg)

jacCER(z0, dimx, dimlam, heobj=heobj,
argheobj=myarg, constr=g, grconstr=grg, heconstr=heg)
```

compl *Complementarity functions*

Description

Classic Complementarity functions

Usage

```
phiFB(a, b)
GrAphiFB(a, b)
GrBphiFB(a, b)

phipFB(a, b, p)
GrAphipFB(a, b, p)
GrBphipFB(a, b, p)

phirFB(a, b)
GrAphirFB(a, b)
GrBphirFB(a, b)

phiMin(a, b)
GrAphiMin(a, b)
GrBphiMin(a, b)

phiMan(a, b, f, fprime)
GrAphiMan(a, b, f, fprime)
GrBphiMan(a, b, f, fprime)

phiKK(a, b, lambda)
GrAphiKK(a, b, lambda)
GrBphiKK(a, b, lambda)


phiLT(a, b, q)
GrAphiLT(a, b, q)
GrBphiLT(a, b, q)

compl.par(type=c("FB", "pFB", "rFB", "Min", "Man", "LT", "KK"),
p, f, fprime, q, lambda)
```

```
## S3 method for class 'compl.par'
print(x, ...)

## S3 method for class 'compl.par'
summary(object, ...)
```

Arguments

a	first parameter.
b	second parameter.
f, fprime	a univariate function and its derivative.
lambda	a parameter in [0, 2[.
q	a parameter >1.
p	a parameter >0.
type	a character string for the complementarity function type: either "FB", "Min", "Man", "LT" or "KK".
x, object	an object of class "compl.par".
...	further arguments to pass to print, or summary.

Details

We implement 5 complementarity functions From Facchinei & Pang (2003).

(i) phiFB the Fischer-Burmeister complementarity function $\sqrt{a^2+b^2}-(a+b)$. The penalized version is $phiFB(a,b) - p * max(a,0) * max(b,0)$, whereas the regularized version is $phiFB(a,b) - epsilon$.

(ii) phiMin the minimum complementarity function $\min(a,b)$.

(iii) phiMan the Mangasarian's family of complementarity function $f(|a-b|)-f(a)-f(b)$, typically $f(t)=t$ or $f(t)=t^3$.

(iv) phiKK the Kanzow-Kleinmichel complementarity function $(\sqrt{((a-b)^2+2*\lambda*a*b)}-(a+b))/(2-\lambda)$.

(v) phiLT the Luo-Tseng complementarity function $(a^q+b^q)^(1/q)-(a+b)$.

GrAXXX and GrBXXX implements the derivative of the complementarity function XXX with respect to a and b respectively.

compl.par creates an object of class "compl.par" with attributes "type" a character string and "fun","grA","grB" the corresponding functions for a given type. Optional arguments are also available, e.g. lambda for the KK complementarity function.

Value

A numeric or an object of class "compl.par".

Author(s)

Christophe Dutang

References

F. Facchinei and J.S. Pang, *Finite-Dimensional Variational Inequalities and Complementarity Problems*, Springer-Verlag (New York 2003).

See Also

See also GNE.nseq.

Examples

```
phiFB(1, 2)
phiLT(1, 2, 2)
phiKK(1, 2, 1)

-2*phiMin(1, 2)
phiMan(1, 2, function(t) t)

complFB <- compl.par("FB")
summary(complFB)

complKK <- compl.par("KK", lambda=1)
summary(complKK)

complKK$fun(1, 1, complKK$lambda)
complFB$fun(1, 1)
```

`eqsolve` *Solving non linear equations*

Description

Non linear Solving methods

Usage

```
eqsolve(xinit, f, jac,
    method=c("Newton", "Levenberg-Marquardt", "Broyden"),
global=c("line search", "none"), control=list())
```

Arguments

`xinit`	initial point.
`f`	the function for which we search roots.
`jac`	the Jacobian of the function `f`.
`method`	a character string specifying the method to use: either `"Newton"`, `"Levenberg-Marquardt"`, or `"Broyden"`.
`global`	a character string for the globalization method to be used: either `"line search"` or `"none"`.
`control`	a list for the control parameters. See details.

Details

The `control` argument is a list that can supply any of the following components:

`tol` The absolute convergence tolerance. Default to 1e-6.

`maxit` The maximum number of iterations. Default to 100.

`echo` A logical or an integer (0, 1, 2, 3, 4) to print traces. Default to `FALSE`, i.e. 0.

`echofile` A character string to store the traces in that file. Default to `NULL`.

`echograph` A character string to plot iter-by-iter information. Either `"NULL"` (default), or `"line"` for line search plot or `"trust"` for trust region plots.

`sigma` Reduction factor for the geometric linesearch. Default to 0.5.

`btol` The backtracking tolerance. Default to 0.01.

`delta` The exponent parameter for the LM parameter, should in $[1, 2]$. Default to 2.

`initlnsrch` The initial integer for starting the line search. Default to 0.

`minstep` The minimal step. Default to 0.001.

Value

A list with components:

`par` The best set of parameters found.

`counts` A two-element integer vector giving the number of calls to `phi` and `jacphi` respectively.

`iter` The iteration number.

`code` 0 if convergence, 1 if `maxit` is reached, 10 if `tol` is not reached and 11 for both.

Author(s)

Christophe Dutang

See Also

See `nleqslv` from the package of the same name.

GNE *GNE package*

Description

Generalized Nash Equilibrium computational methods.

Usage

```
GNE(approach =
c("non smooth", "fixed point", "minimization", "constrained equation"),
method = "default", xinit, control=list(), ...)
```

Arguments

approach	a character string for the approach: either "non smooth", "fixed point", "minimization" or "constrained equation".
method	a character string for the computation method: either "default" or the name of the method.
xinit	a numeric vector for the initial point.
...	further arguments to be passed to GNE.nseq, GNE.fpeq or GNE.minpb.
control	a list with control parameters.

Details

Computing generalized Nash Equilibrium can be done in three different approaches.

(i) extended KKT system It consists in solving the non smooth extended Karush-Kuhn-Tucker (KKT) system $\Phi(z) = 0$.

(ii) fixed point approach It consists in solving equation $y(x) = x$.

(iii) gap function minimization It consists in minimizing a gap function $minV(x)$.

(iv) constrained equation It consists in solving $F(x)$ such that x belongs to a specific set.

The GNE function is a global function calling the appropriate function GNE.nseq, GNE.fpeq, GNE.ceq or GNE.minpb. Benchmark functions comparing all methods for a given reformulation are available: see bench.GNE.

Additionnal utitilty functions are also available: rejection, projector, stepfunc, complementarity and funSSR.

Value

A list with components:

`par` The best set of parameters found.

`value` The value of the merit function.

`counts` A two-element integer vector giving the number of calls to `phi` and `jacphi` respectively.

`iter` The outer iteration number.

`code` The values returned are

- `1` Function criterion is near zero. Convergence of function values has been achieved.
- `2` x-values within tolerance. This means that the relative distance between two consecutive x-values is smaller than `xtol`.
- `3` No better point found. This means that the algorithm has stalled and cannot find an acceptable new point. This may or may not indicate acceptably small function values.
- `4` Iteration limit `maxit` exceeded.
- `5` Jacobian is too ill-conditioned.
- `6` Jacobian is singular.
- `100` an error in the execution.

`message` a string describing the termination code

`fvec` a vector with function values.

`approach` the name of the approach.

Author(s)

Christophe Dutang

References

F. Facchinei, A. Fischer and V. Piccialli (2009), *Generalized Nash equilibrium problems and Newton methods*, Math. Program.

A. von Heusinger (2009), *Numerical Methods for the Solution of the Generalized Nash Equilibrium Problem*, Ph. D. Thesis.

A. von Heusinger and C. Kanzow (2009), *Optimization reformulations of the generalized Nash equilibrium problem using Nikaido-Isoda-type functions*, Comput Optim Appl .

F. Facchinei and C. Kanzow (2009), *Generalized Nash Equilibrium problems.* Preprint 290.

C. Dutang (2013), *A survey of GNE computation methods: theory and algorithms*, preprint on HAL, https://hal.science/hal-00813531.

See Also

See `GNE.fpeq`, `GNE.minpb`, `GNE.ceq` and `GNE.nseq` for other approaches.

GNE.ceq *Constrained equation reformulation of the GNE problem.*

Description

Constrained equation reformulation via the extended KKT system of the GNE problem.

Usage

```
GNE.ceq(init, dimx, dimlam, grobj, arggrobj, heobj, argheobj,
constr, argconstr, grconstr, arggrconstr, heconstr, argheconstr,
dimmu, joint, argjoint, grjoint, arggrjoint, hejoint, arghejoint,
method="PR", control=list(), silent=TRUE, ...)
```

Arguments

init	Initial values for the parameters to be optimized over: $z = (x, lambda, mu)$.
dimx	a vector of dimension for x.
dimlam	a vector of dimension for $lambda$.
grobj	gradient of the objective function (to be minimized), see details.
arggrobj	a list of additional arguments of the objective gradient.
heobj	Hessian of the objective function, see details.
argheobj	a list of additional arguments of the objective Hessian.
constr	constraint function ($g^i(x) <= 0$), see details.
argconstr	a list of additional arguments of the constraint function.
grconstr	gradient of the constraint function, see details.
arggrconstr	a list of additional arguments of the constraint gradient.
heconstr	Hessian of the constraint function, see details.
argheconstr	a list of additional arguments of the constraint Hessian.
dimmu	a vector of dimension for mu.
joint	joint function ($h(x) <= 0$), see details.
argjoint	a list of additional arguments of the joint function.
grjoint	gradient of the joint function, see details.
arggrjoint	a list of additional arguments of the joint gradient.
hejoint	Hessian of the joint function, see details.
arghejoint	a list of additional arguments of the joint Hessian.
method	a character string specifying the method "PR" or "AS".
control	a list with control parameters.
...	further arguments to be passed to the optimization routine. NOT to the functions H and jacH.
silent	a logical to get some traces. Default to FALSE.

Details

GNE.ceq solves the GNE problem via a constrained equation reformulation of the KKT system.

This approach consists in solving the extended Karush-Kuhn-Tucker (KKT) system denoted by $H(z) = 0$, for $z \in \Omega$ where z is formed by the players strategy x, the Lagrange multiplier λ and the slate variable w. The root problem $H(z) = 0$ is solved by an iterative scheme $z_{n+1} = z_n + d_n$, where the direction d_n is computed in two different ways. Let $J(x) = JacH(x)$. There are two possible methods either "PR" for potential reduction algorithm or "AS" for affine scaled trust reduction algorithm.

(a) potential reduction algorithm: The direction solves the system $H(z_n)+J(z_n)d = sigma_n a^T H(z_n)/||a||_2^2 a$.

(b) bound-constrained trust region algorithm: The direction solves the system $\min_p ||J(z_n)^T p+ H(z_n)||^2$, for p such that $||p|| <= Delta_n||$.

... are further arguments to be passed to the optimization routine, that is global, xscalm, silent. A globalization scheme can be choosed using the global argument. Available schemes are

(1) Line search: if global is set to "qline" or "gline", a line search is used with the merit function being half of the L2 norm of Phi, respectively with a quadratic or a geometric implementation.

(3) Trust-region: if global is set to "pwldog", the Powell dogleg method is used.

(2) None: if global is set to "none", no globalization is done.

The default value of global is "gline" when method="PR" and "pwldog" when method="AS". The xscalm is a scaling parameter to used, either "fixed" (default) or "auto", for which scaling factors are calculated from the euclidean norms of the columns of the jacobian matrix. The silent argument is a logical to report or not the optimization process, default to FALSE.

The control argument is a list that can supply any of the following components:

xtol The relative steplength tolerance. When the relative steplength of all scaled x values is smaller than this value convergence is declared. The default value is 10^{-8}.

ftol The function value tolerance. Convergence is declared when the largest absolute function value is smaller than ftol. The default value is 10^{-8}.

btol The backtracking tolerance. The default value is 10^{-2}.

maxit The maximum number of major iterations. The default value is 100 if a global strategy has been specified.

trace Non-negative integer. A value of 1 will give a detailed report of the progress of the iteration, default 0.

sigma, delta, zeta Parameters initialized to 1/2, 1, length(init)/2, respectively, when method="PR".

forcingpar Forcing parameter set to 0.1, when method="PR".

theta, radiusmin, reducmin, radiusmax, radiusred, reducred, radiusexp, reducexp Parameters initialized to 0.99995, 1, 0.1, 1e10, 1/2, 1/4, 2, 3/4, when method="AS".

Value

GNE.ceq returns a list with components:

par The best set of parameters found.

value The value of the merit function.

counts A two-element integer vector giving the number of calls to H and jacH respectively.

iter The outer iteration number.

code The values returned are

- 1 Function criterion is near zero. Convergence of function values has been achieved.
- 2 x-values within tolerance. This means that the relative distance between two consecutive x-values is smaller than xtol.
- 3 No better point found. This means that the algorithm has stalled and cannot find an acceptable new point. This may or may not indicate acceptably small function values.
- 4 Iteration limit maxit exceeded.
- 5 Jacobian is too ill-conditioned.
- 6 Jacobian is singular.
- 100 an error in the execution.

message a string describing the termination code.

fvec a vector with function values.

Author(s)

Christophe Dutang

References

J.E. Dennis and J.J. Moree (1977), *Quasi-Newton methods, Motivation and Theory*, SIAM review.

Monteiro, R. and Pang, J.-S. (1999), *A Potential Reduction Newton Method for Constrained equations*, SIAM Journal on Optimization 9(3), 729-754.

S. Bellavia, M. Macconi and B. Morini (2003), *An affine scaling trust-region approach to bound-constrained nonlinear systems*, Applied Numerical Mathematics 44, 257-280

A. Dreves, F. Facchinei, C. Kanzow and S. Sagratella (2011), *On the solutions of the KKT conditions of generalized Nash equilibrium problems*, SIAM Journal on Optimization 21(3), 1082-1108.

See Also

See GNE.fpeq, GNE.minpb and GNE.nseq for other approaches; funCER and jacCER for template functions of H and $JacH$.

Examples

```
#-------------------------------------------------------------------------
# (1) Example 5 of von Facchinei et al. (2007)
#-------------------------------------------------------------------------
```

```
dimx <- c(1, 1)
#Gr_x_j O_i(x)
grobj <- function(x, i, j)
{
if(i == 1)
res <- c(2*(x[1]-1), 0)
if(i == 2)
res <- c(0, 2*(x[2]-1/2))
res[j]
}
#Gr_x_k Gr_x_j O_i(x)
heobj <- function(x, i, j, k)
2 * (i == j && j == k)

dimlam <- c(1, 1)
#constraint function g_i(x)
g <- function(x, i)
sum(x[1:2]) - 1
#Gr_x_j g_i(x)
grg <- function(x, i, j)
1
#Gr_x_k Gr_x_j g_i(x)
heg <- function(x, i, j, k)
0


x0 <- rep(0, sum(dimx))
z0 <- c(x0, 2, 2, max(10, 5-g(x0, 1) ), max(10, 5-g(x0, 2) ) )

#true value is (3/4, 1/4, 1/2, 1/2)
GNE.ceq(z0, dimx, dimlam, grobj=grobj, heobj=heobj,
constr=g, grconstr=grg, heconstr=heg, method="PR",
control=list(trace=0, maxit=10))


GNE.ceq(z0, dimx, dimlam, grobj=grobj, heobj=heobj,
constr=g, grconstr=grg, heconstr=heg, method="AS", global="pwldog",
xscalm="auto", control=list(trace=0, maxit=100))


#------------------------------------------------------------------------------
# (2) Duopoly game of Krawczyk and Stanislav Uryasev (2000)
#------------------------------------------------------------------------------


#constants
myarg <- list(d= 20, lambda= 4, rho= 1)

dimx <- c(1, 1)
#Gr_x_j O_i(x)
grobj <- function(x, i, j, arg)
{
```

```
res <- -arg$rho * x[i]
if(i == j)
res <- res + arg$d - arg$lambda - arg$rho*(x[1]+x[2])
-res
}
#Gr_x_k Gr_x_j O_i(x)
heobj <- function(x, i, j, k, arg)
arg$rho * (i == j) + arg$rho * (j == k)


dimlam <- c(1, 1)
#constraint function g_i(x)
g <- function(x, i)
-x[i]
#Gr_x_j g_i(x)
grg <- function(x, i, j)
-1*(i == j)
#Gr_x_k Gr_x_j g_i(x)
heg <- function(x, i, j, k)
0

#true value is (16/3, 16/3, 0, 0)

x0 <- rep(0, sum(dimx))
z0 <- c(x0, 2, 2, max(10, 5-g(x0, 1) ), max(10, 5-g(x0, 2) ) )


GNE.ceq(z0, dimx, dimlam, grobj=grobj, heobj=heobj, arggrobj=myarg,
argheobj=myarg, constr=g, grconstr=grg, heconstr=heg,
method="PR", control=list(trace=0, maxit=10))

GNE.ceq(z0, dimx, dimlam, grobj=grobj, heobj=heobj, arggrobj=myarg,
argheobj=myarg, constr=g, grconstr=grg, heconstr=heg,
method="AS", global="pwldog", xscalm="auto", control=list(trace=0, maxit=100))
```

GNE.fpeq *Fixed point equation reformulation of the GNE problem.*

Description

Fixed point equation reformulation via the NI function of the GNE problem.

Usage

```
GNE.fpeq(init, dimx, obj, argobj, grobj, arggrobj,
heobj, argheobj, joint, argjoint, jacjoint, argjacjoint,
method = "default", problem = c("NIR", "VIR"),
```

```
merit = c("NI", "VI", "FP"), order.method=1, control.outer=list(),
control.inner=list(), silent=TRUE, param=list(), stepfunc, argstep, ...)
```

Arguments

init	Initial values for the parameters to be optimized over: $z = (x, lambda, mu)$.
dimx	a vector of dimension for x.
obj	objective function (to be minimized), see details.
argobj	a list of additional arguments.
grobj	gradient of the objective function, see details.
arggrobj	a list of additional arguments of the objective gradient.
heobj	Hessian of the objective function, see details.
argheobj	a list of additional arguments of the objective Hessian.
joint	joint function ($h(x) <= 0$), see details.
argjoint	a list of additional arguments of the joint function.
jacjoint	Jacobian of the joint function, see details.
argjacjoint	a list of additional arguments of the Jacobian.
method	either "pure", "UR", "vH", "RRE", "MPE", "SqRRE" or "SqMPE" method, see details. "default" corresponds to "MPE".
problem	either "NIR", "VIP", see details.
merit	either "NI", "VI", "FP", see details.
order.method	the order of the extrapolation method.
control.outer	a list with control parameters for the fixed point algorithm.
control.inner	a list with control parameters for the fixed point function.
silent	a logical to show some traces.
param	a list of parameters for the computation of the fixed point function.
stepfunc	the step function, only needed when method="UR".
argstep	additional arguments for the step function.
...	further arguments to be passed to the optimization routine. NOT to the functions.

Details

Functions in argument must respect the following template:

- obj must have arguments the current iterate z, the player number i and optionnally additional arguments given in a list.
- grobj must have arguments the current iterate z, the player number i, the derivative index j and optionnally additional arguments given in a list.
- heobj must have arguments the current iterate z, the player number i, the derivative indexes j, k and optionnally additional arguments given in a list.
- joint must have arguments the current iterate z and optionnally additional arguments given in a list.

- `jacjoint` must have arguments the current iterate `z`, the derivative index `j` and optionnally additional arguments given in a list.

The fixed point approach consists in solving equation $y(x) = x$.

(a) Crude or pure fixed point method: It simply consists in iterations $x_{n+1} = y(x_n)$.

(b) Polynomial methods: - relaxation algorithm (linear extrapolation): The next iterate is computed as

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n y(x_n).$$

The step α_n can be computed in different ways: constant, decreasing serie or a line search method. In the literature of game theory, the decreasing serie refers to the method of Ursayev and Rubinstein (`method="UR"`) while the line search method refers to the method of von Heusinger (`method="vH"`). Note that the constant step can be done using the UR method.

- RRE and MPE method: Reduced Rank Extrapolation and Minimal Polynomial Extrapolation methods are polynomial extrapolation methods, where the monomials are functional "powers" of the y function, i.e. function composition of y. Of order 1, RRE and MPE consists of

$$x_{n+1} = x_n + t_n(y(x_n) - x_n),$$

where t_n equals to $< v_n, r_n > / < v_n, v_n >$ for RRE1 and $< r_n, r_n > / < v_n, r_n >$ for MPE1, where $r_n = y(x_n) - x_n$ and $v_n = y(y(x_n)) - 2y(x_n) + x_n$. To use RRE/MPE methods, set `method = "RRE"` or `method = "MPE"`.

- squaring method: It consists in using an extrapolation method (such as RRE and MPE) after two iteration of the linear extrapolation, i.e.

$$x_{n+1} = x_n - 2t_n r_n + t_n^2 v_n.$$

The squared version of RRE/MPE methods are available via setting `method = "SqRRE"` or `method = "SqMPE"`.

(c) Epsilon algorithms: Not implemented.

For details on fixed point methods, see Varadhan & Roland (2004).

The `control.outer` argument is a list that can supply any of the following components:

`merit="FP"` **and** `method="pure"` see `fpiter`. the default parameters are `list(tol=1e-6, maxiter=100, trace=TRUE)`.

`merit="FP"` **and** `method!="pure"` see `squarem`. the default parameters are `list(tol=1e-6, maxiter=100, trace=TRUE)`.

`merit!="FP"` parameters are

`tol` The absolute convergence tolerance. Default to 1e-6.

`maxit` The maximum number of iterations. Default to 100.

`echo` A logical or an integer (0, 1, 2, 3) to print traces. Default to `FALSE`, i.e. 0.

`sigma, beta` parameters for von Heusinger algorithm. Default to 9/10 and 1/2 respectively.

Value

A list with components:

`par` The best set of parameters found.

`value` The value of the merit function.

`outer.counts` A two-element integer vector giving the number of calls to fixed-point and merit functions respectively.

`outer.iter` The outer iteration number.

`code` The values returned are

- `1` Function criterion is near zero. Convergence of function values has been achieved.
- `4` Iteration limit `maxit` exceeded.
- `100` an error in the execution.

`inner.iter` The iteration number when computing the fixed-point function.

`inner.counts` A two-element integer vector giving the number of calls to the gap function and its gradient when computing the fixed-point function.

`message` a string describing the termination code

Author(s)

Christophe Dutang

References

A. von Heusinger (2009), *Numerical Methods for the Solution of the Generalized Nash Equilibrium Problem*, Ph. D. Thesis.

A. von Heusinger and C. Kanzow (2009), *Optimization reformulations of the generalized Nash equilibrium problem using Nikaido-Isoda-type functions*, Comput Optim Appl .

S. Uryasev and R.Y. Rubinstein (1994), *On relaxation algorithms in computation of noncooperative equilibria*, IEEE Transactions on Automatic Control.

R. Varadhan and C. Roland (2004), *Squared Extrapolation Methods (SQUAREM): A New Class of Simple and Efficient Numerical Schemes for Accelerating the Convergence of the EM Algorithm*, Johns Hopkins University, Dept. of Biostatistics Working Papers.

See Also

See `GNE.ceq`, `GNE.minpb` and `GNE.nseq` for other approaches.

GNE.minpb *Non smooth equation reformulation of the GNE problem.*

Description

Non smooth equation reformulation via the extended KKT system of the GNE problem.

Usage

```
GNE.minpb(init, dimx, obj, argobj, grobj, arggrobj,
heobj, argheobj, joint, argjoint, jacjoint, argjacjoint,
method="default", problem = c("NIR", "VIR"), control.outer=list(),
control.inner=list(), silent=TRUE, param=list(),
optim.type=c("free","constr"), ...)
```

Arguments

init	Initial values for the parameters to be optimized over: $z = (x, lambda, mu)$.
dimx	a vector of dimension for x.
obj	objective function (to be minimized), see details.
argobj	a list of additional arguments.
grobj	gradient of the objective function, see details.
arggrobj	a list of additional arguments of the objective gradient.
heobj	Hessian of the objective function, see details.
argheobj	a list of additional arguments of the objective Hessian.
joint	joint function ($h(x) <= 0$), see details.
argjoint	a list of additional arguments of the joint function.
jacjoint	Jacobian of the joint function, see details.
argjacjoint	a list of additional arguments of the Jacobian.
method	either "BB", "CG" or "BFGS", see details.
problem	either "NIR", "VIP", see details.
optim.type	either "free", "constr", see details.
control.outer	a list with control parameters for the minimization algorithm.
control.inner	a list with control parameters for the minimization function.
...	further arguments to be passed to the optimization routine. NOT to the functions phi and jacphi.
silent	a logical to show some traces.
param	a list of parameters for the computation of the minimization function.

Details

Functions in argument must respect the following template:

- `obj` must have arguments the current iterate `z`, the player number `i` and optionnally additional arguments given in a list.
- `grobj` must have arguments the current iterate `z`, the player number `i`, the derivative index `j` and optionnally additional arguments given in a list.
- `heobj` must have arguments the current iterate `z`, the player number `i`, the derivative indexes `j`, `k` and optionnally additional arguments given in a list.
- `joint` must have arguments the current iterate `z` and optionnally additional arguments given in a list.
- `jacjoint` must have arguments the current iterate `z`, the derivative index `j` and optionnally additional arguments given in a list.

The gap function minimization consists in minimizing a gap function $minV(x)$. The function `minGap` provides two optimization methods to solve this minimization problem.

Barzilai-Borwein algorithm when `method = "BB"`, we use Barzilai-Borwein iterative scheme to find the minimum.

Conjugate gradient algorithm when `method = "CG"`, we use the CG iterative scheme implemented in `R`, an Hessian-free method.

Broyden-Fletcher-Goldfarb-Shanno algorithm when `method = "BFGS"`, we use the BFGS iterative scheme implemented in `R`, a quasi-Newton method with line search.

In the game theory literature, there are two main gap functions: the regularized Nikaido-Isoda (NI) function and the regularized QVI gap function. This correspond to `type="NI"` and `type="VI"`, respectively. See von Heusinger & Kanzow (2009) for details on the NI function and Kubota & Fukushima (2009) for the QVI regularized gap function.

The `control.outer` argument is a list that can supply any of the following components:

`tol` The absolute convergence tolerance. Default to 1e-6.

`maxit` The maximum number of iterations. Default to 100.

`echo` A logical or an integer (0, 1, 2, 3) to print traces. Default to `FALSE`, i.e. 0.

`stepinit` Initial step size for the BB method (should be small if gradient is "big"). Default to 1.

Note that the `Gap` function can return a numeric or a list with computation details. In the latter case, the object return must be a list with the following components `value`, `counts`, `iter`, see the example below.

Value

A list with components:

`par` The best set of parameters found.

`value` The value of the merit function.

`outer.counts` A two-element integer vector giving the number of calls to `Gap` and `gradGap` respectively.

`outer.iter` The outer iteration number.

`code` The values returned are

- `1` Function criterion is near zero. Convergence of function values has been achieved.
- `2` x-values within tolerance. This means that the relative distance between two consecutive x-values is smaller than `xtol`.
- `3` No better point found. This means that the algorithm has stalled and cannot find an acceptable new point. This may or may not indicate acceptably small function values.
- `4` Iteration limit `maxit` exceeded.
- `5` Jacobian is too ill-conditioned.
- `6` Jacobian is singular.
- `100` an error in the execution.

`inner.iter` The iteration number when computing the minimization function.

`inner.counts` A two-element integer vector giving the number of calls to the gap function and its gradient when computing the minimization function.

`message` a string describing the termination code

Author(s)

Christophe Dutang

References

A. von Heusinger (2009), *Numerical Methods for the Solution of the Generalized Nash Equilibrium Problem*, Ph. D. Thesis.

A. von Heusinger and C. Kanzow (2009), *Optimization reformulations of the generalized Nash equilibrium problem using Nikaido-Isoda-type functions*, Comput Optim Appl .

K. Kubota and M. Fukushima (2009), *Gap function approach to the generalized Nash Equilibrium problem*, Journal of Optimization theory and applications.

See Also

See `GNE.fpeq`, `GNE.ceq` and `GNE.nseq` for other approaches.

`GNE.nseq` *Non smooth equation reformulation of the GNE problem.*

Description

Non smooth equation reformulation via the extended KKT system of the GNE problem.

Usage

```
GNE.nseq(init, dimx, dimlam, grobj, arggrobj, heobj, argheobj,
constr, argconstr, grconstr, arggrconstr, heconstr, argheconstr,
compl, gcompla, gcomplb, argcompl,
dimmu, joint, argjoint, grjoint, arggrjoint, hejoint, arghejoint,
method="default", control=list(), silent=TRUE, ...)
```

Arguments

`init`	Initial values for the parameters to be optimized over: $z = (x, lambda, mu)$.
`dimx`	a vector of dimension for x.
`dimlam`	a vector of dimension for $lambda$.
`grobj`	gradient of the objective function (to be minimized), see details.
`arggrobj`	a list of additional arguments of the objective gradient.
`heobj`	Hessian of the objective function, see details.
`argheobj`	a list of additional arguments of the objective Hessian.
`constr`	constraint function ($g^i(x) <= 0$), see details.
`argconstr`	a list of additional arguments of the constraint function.
`grconstr`	gradient of the constraint function, see details.
`arggrconstr`	a list of additional arguments of the constraint gradient.
`heconstr`	Hessian of the constraint function, see details.
`argheconstr`	a list of additional arguments of the constraint Hessian.
`compl`	the complementarity function with (at least) two arguments: `compl(a,b)`.
`argcompl`	list of possible additional arguments for `compl`.
`gcompla`	derivative of the complementarity function w.r.t. the first argument.
`gcomplb`	derivative of the complementarity function w.r.t. the second argument.
`dimmu`	a vector of dimension for mu.
`joint`	joint function ($h(x) <= 0$), see details.
`argjoint`	a list of additional arguments of the joint function.
`grjoint`	gradient of the joint function, see details.
`arggrjoint`	a list of additional arguments of the joint gradient.
`hejoint`	Hessian of the joint function, see details.
`arghejoint`	a list of additional arguments of the joint Hessian.
`method`	a character string specifying the method `"Newton"`, `"Broyden"`, `"Levenberg-Marquardt"` or `"default"` which is `"Newton"`.
`control`	a list with control parameters.
`...`	further arguments to be passed to the optimization routine. NOT to the functions `phi` and `jacphi`.
`silent`	a logical to get some traces. Default to `FALSE`.

Details

Functions in argument must respect the following template:

- `constr` must have arguments the current iterate `z`, the player number `i` and optionnally additional arguments given in a list.
- `grobj`, `grconstr` must have arguments the current iterate `z`, the player number `i`, the derivative index `j` and optionnally additional arguments given in a list.

- heobj, heconstr must have arguments the current iterate z, the player number i, the derivative indexes j, k and optionnally additional arguments given in a list.
- compl, gcompla, gcomplb must have two arguments a, b and optionnally additional arguments given in a list.
- joint must have arguments the current iterate z and optionnally additional arguments given in a list.
- grjoint must have arguments the current iterate z, the derivative index j and optionnally additional arguments given in a list.
- hejoint must have arguments the current iterate z, the derivative indexes j, k and optionnally additional arguments given in a list.

GNE.nseq solves the GNE problem via a non smooth reformulation of the KKT system. bench.GNE.nseq carries out a benchmark of the computation methods (Newton and Broyden direction with all possible global schemes) for a given initial point. bench.GNE.nseq.LM carries out a benchmark of the Levenberg-Marquardt computation method.

This approach consists in solving the extended Karush-Kuhn-Tucker (KKT) system denoted by $\Phi(z) = 0$, where z is formed by the players strategy x and the Lagrange multiplier λ. The root problem $\Phi(z) = 0$ is solved by an iterative scheme $z_{n+1} = z_n + d_n$, where the direction d_n is computed in three different ways. Let $J(x) = Jac\Phi(x)$.

(a) Newton: The direction solves the system $J(z_n)d = -\Phi(z_n)$, generally called the Newton equation.

(b) Broyden: It is a quasi-Newton method aiming to solve an approximate version of the Newton equation $d = -\Phi(z_n)W_n$ where W_n is computed by an iterative scheme. In the current implementation, W_n is updated by the Broyden method.

(c) Levenberg-Marquardt: The direction solves the system

$$\left[J(z_n)^T J(z_n) + \lambda_n^\delta I\right] d = -J(z_n)^T \Phi(x_n)$$

where I denotes the identity matrix, δ is a parameter in [1,2] and $\lambda_n = ||\Phi(z_n)||$ if LM.param="merit", $||J(z_n)^T\Phi(z_n)||$ if LM.param="jacmerit", the minimum of both preceding quantities if LM.param="min", or an adatpive parameter according to Fan(2003) if LM.param="adaptive".

In addition to the computation method, a globalization scheme can be choosed using the global argument, via the ... argument. Available schemes are

(1) Line search: if global is set to "qline" or "gline", a line search is used with the merit function being half of the L2 norm of Phi, respectively with a quadratic or a geometric implementation.

(2) Trust region: if global is set to "dbldog" or "pwldog", a trust region is used respectively with a double dogleg or a Powell (simple) dogleg implementation. This global scheme is not available for the Levenberg-Marquardt direction.

(3) None: if global is set to "none", no globalization is done.

The default value of global is "gline". Note that in the special case of the Levenberg-Marquardt direction with adaptive parameter, the global scheme must be "none".

In the GNEP context, details on the methods can be found in Facchinei, Fischer & Piccialli (2009), `"Newton"` corresponds to method 1 and `"Levenberg-Marquardt"` to method 3. In a general non-linear equation framework, see Dennis & Moree (1977), Dennis & Schnabel (1996) or Nocedal & Wright (2006),

The implementation relies heavily on the `nleqslv` function of the package of the same name. So full details on the control parameters are to be found in the help page of this function. We briefly recall here the main parameters. The `control` argument is a list that can supply any of the following components:

`xtol` The relative steplength tolerance. When the relative steplength of all scaled x values is smaller than this value convergence is declared. The default value is 10^{-8}.

`ftol` The function value tolerance. Convergence is declared when the largest absolute function value is smaller than `ftol`. The default value is 10^{-8}.

`delta` A numeric `delta` in [1, 2], default to 2, for the Levenberg-Marquardt method only.

`LM.param` A character string, default to `"merit"`, for the Levenberg-Marquardt method only.

`maxit` The maximum number of major iterations. The default value is 150 if a global strategy has been specified.

`trace` Non-negative integer. A value of 1 will give a detailed report of the progress of the iteration, default 0.

`...` are further arguments to be passed to the optimization routine, that is `global`, `xscalm`, `silent`. See above for the globalization scheme. The `xscalm` is a scaling parameter to used, either `"fixed"` (default) or `"auto"`, for which scaling factors are calculated from the euclidean norms of the columns of the jacobian matrix. See `nleqslv` for details. The `silent` argument is a logical to report or not the optimization process, default to `FALSE`.

Value

`GNE.nseq` returns a list with components:

`par` The best set of parameters found.

`value` The value of the merit function.

`counts` A two-element integer vector giving the number of calls to `phi` and `jacphi` respectively.

`iter` The outer iteration number.

`code` The values returned are

- `1` Function criterion is near zero. Convergence of function values has been achieved.
- `2` x-values within tolerance. This means that the relative distance between two consecutive x-values is smaller than `xtol`.
- `3` No better point found. This means that the algorithm has stalled and cannot find an acceptable new point. This may or may not indicate acceptably small function values.
- `4` Iteration limit `maxit` exceeded.
- `5` Jacobian is too ill-conditioned.
- `6` Jacobian is singular.
- `100` an error in the execution.

`message` a string describing the termination code.

`fvec` a vector with function values.

`bench.GNE.nseq` returns a list with components:

`compres` a data.frame summarizing the different computations.

`reslist` a list with the different results from `GNE.nseq`.

Author(s)

Christophe Dutang

References

J.E. Dennis and J.J. Moree (1977), *Quasi-Newton methods, Motivation and Theory*, SIAM review.

J.E. Dennis and R.B. Schnabel (1996), *Numerical methods for unconstrained optimization and nonlinear equations*, SIAM.

F. Facchinei, A. Fischer and V. Piccialli (2009), *Generalized Nash equilibrium problems and Newton methods*, Math. Program.

J.-Y. Fan (2003), *A modified Levenberg-Marquardt algorithm for singular system of nonlinear equations*, Journal of Computational Mathematics.

B. Hasselman (2011), *nleqslv: Solve systems of non linear equations*, R package.

A. von Heusinger and C. Kanzow (2009), *Optimization reformulations of the generalized Nash equilibrium problem using Nikaido-Isoda-type functions*, Comput Optim Appl .

J. Nocedal and S.J. Wright (2006), *Numerical Optimization*, Springer Science+Business Media

See Also

See `GNE.fpeq`, `GNE.ceq` and `GNE.minpb` for other approaches; `funSSR` and `jacSSR` for template functions of Φ and $Jac\Phi$ and `complementarity` for complementarity functions.

See also `nleqslv` for some optimization details.

Examples

```
#-------------------------------------------------------------------------------
# (1) Example 5 of von Facchinei et al. (2007)
#-------------------------------------------------------------------------------

dimx <- c(1, 1)
#Gr_x_j O_i(x)
grobj <- function(x, i, j)
{
if(i == 1)
res <- c(2*(x[1]-1), 0)
if(i == 2)
res <- c(0, 2*(x[2]-1/2))
res[j]
}
#Gr_x_k Gr_x_j O_i(x)
```

```
heobj <- function(x, i, j, k)
2 * (i == j && j == k)

dimlam <- c(1, 1)
#constraint function g_i(x)
g <- function(x, i)
sum(x[1:2]) - 1
#Gr_x_j g_i(x)
grg <- function(x, i, j)
1
#Gr_x_k Gr_x_j g_i(x)
heg <- function(x, i, j, k)
0



#true value is (3/4, 1/4, 1/2, 1/2)

z0 <- rep(0, sum(dimx)+sum(dimlam))

funSSR(z0, dimx, dimlam, grobj=grobj, constr=g, grconstr=grg, compl=phiFB, echo=FALSE)


jacSSR(z0, dimx, dimlam, heobj=heobj, constr=g, grconstr=grg,
heconstr=heg, gcompla=GrAphiFB, gcomplb=GrBphiFB)


GNE.nseq(z0, dimx, dimlam, grobj=grobj, NULL, heobj=heobj, NULL,
constr=g, NULL, grconstr=grg, NULL, heconstr=heg, NULL,
compl=phiFB, gcompla=GrAphiFB, gcomplb=GrBphiFB, method="Newton",
control=list(trace=1))

GNE.nseq(z0, dimx, dimlam, grobj=grobj, NULL, heobj=heobj, NULL,
constr=g, NULL, grconstr=grg, NULL, heconstr=heg, NULL,
compl=phiFB, gcompla=GrAphiFB, gcomplb=GrBphiFB, method="Broyden",
control=list(trace=1))



#-----------------------------------------------------------------------------
# (2) Duopoly game of Krawczyk and Stanislav Uryasev (2000)
#-----------------------------------------------------------------------------


#constants
myarg <- list(d= 20, lambda= 4, rho= 1)

dimx <- c(1, 1)
#Gr_x_j O_i(x)
grobj <- function(x, i, j, arg)
{
res <- -arg$rho * x[i]
if(i == j)
```

```
res <- res + arg$d - arg$lambda - arg$rho*(x[1]+x[2])
-res
}
#Gr_x_k Gr_x_j O_i(x)
heobj <- function(x, i, j, k, arg)
arg$rho * (i == j) + arg$rho * (j == k)


dimlam <- c(1, 1)
#constraint function g_i(x)
g <- function(x, i)
-x[i]
#Gr_x_j g_i(x)
grg <- function(x, i, j)
-1*(i == j)
#Gr_x_k Gr_x_j g_i(x)
heg <- function(x, i, j, k)
0

#true value is (16/3, 16/3, 0, 0)

z0 <- rep(0, sum(dimx)+sum(dimlam))

funSSR(z0, dimx, dimlam, grobj=grobj, myarg, constr=g, grconstr=grg, compl=phiFB, echo=FALSE)

jacSSR(z0, dimx, dimlam, heobj=heobj, myarg, constr=g, grconstr=grg,
heconstr=heg, gcompla=GrAphiFB, gcomplb=GrBphiFB)


GNE.nseq(z0, dimx, dimlam, grobj=grobj, myarg, heobj=heobj, myarg,
constr=g, NULL, grconstr=grg, NULL, heconstr=heg, NULL,
compl=phiFB, gcompla=GrAphiFB, gcomplb=GrBphiFB, method="Newton",
control=list(trace=1))

GNE.nseq(z0, dimx, dimlam, grobj=grobj, myarg, heobj=heobj, myarg,
constr=g, NULL, grconstr=grg, NULL, heconstr=heg, NULL,
compl=phiFB, gcompla=GrAphiFB, gcomplb=GrBphiFB, method="Broyden",
control=list(trace=1))
```

NIR *Nikaido Isoda Reformulation*

Description

functions of the Nikaido Isoda Reformulation of the GNEP

Usage

```
gapNIR(x, y, dimx, obj, argobj, param=list(), echo=FALSE)
gradxgapNIR(x, y, dimx, grobj, arggrobj, param=list(), echo=FALSE)
gradygapNIR(x, y, dimx, grobj, arggrobj, param=list(), echo=FALSE)
fpNIR(x, dimx, obj, argobj, joint, argjoint,
grobj, arggrobj, jacjoint, argjacjoint, param=list(),
echo=FALSE, control=list(), yinit=NULL, optim.method="default")
```

Arguments

x, y	a numeric vector.
dimx	a vector of dimension for x.
obj	objective function (to be minimized), see details.
argobj	a list of additional arguments.
grobj	gradient of the objective function, see details.
arggrobj	a list of additional arguments of the objective gradient.
joint	joint function, see details.
argjoint	a list of additional arguments of the joint function.
jacjoint	gradient of the joint function, see details.
argjacjoint	a list of additional arguments of the joint Jacobian.
param	a list of parameters.
control	a list with control parameters for the fixed point algorithm.
yinit	initial point when computing the fixed-point function.
optim.method	optimization method when computing the fixed-point function.
echo	a logical to show some traces.

Details

gapNIR computes the Nikaido Isoda function of the GNEP, while gradxgapNIR and gradygapNIR give its gradient with respect to x and y. fpNIR computes the fixed-point function.

Value

A vector for funSSR or a matrix for jacSSR.

Author(s)

Christophe Dutang

References

A. von Heusinger & J. Kanzow (2009), *Optimization reformulations of the generalized Nash equilibrium problem using Nikaido-Isoda-type functions*, Comput Optim Appl .

F. Facchinei, A. Fischer and V. Piccialli (2009), *Generalized Nash equilibrium problems and Newton methods*, Math. Program.

See Also

See also `GNE.fpeq`.

`potential.reduction` *Potential reduction algorithm utility functions*

Description

Functions for the potential reduction algorithm

Usage

```
potential.ce(u, n, zeta)

gradpotential.ce(u, n, zeta)

psi.ce(z, dimx, dimlam, Hfinal, argfun, zeta)

gradpsi.ce(z, dimx, dimlam, Hfinal, jacHfinal, argfun, argjac, zeta)
```

Arguments

`u`	a numeric vector : $u = (u_1, u_2)$ where u_1 is of size `n`.
`n`	a numeric for the size of u_1.
`zeta`	a positive parameter.
`z`	a numeric vector : $z = (x, lambda, w)$ where `dimx` is the size of components of x and `dimlam` is the size of components of $lambda$ and w.
`dimx`	a numeric vector with the size of each components of x.
`dimlam`	a numeric vector with the size of each components of $lambda$. We must have `length(dimx) == length(dimlam)`.
`Hfinal`	the root function.
`argfun`	a list of additionnals arguments for `Hfinal`.
`jacHfinal`	the Jacobian of the root function.
`argjac`	a list of additionnals arguments for `jacHfinal`.

Details

potential.ce is the potential function for the GNEP, and gradpotential.ce its gradient. psi.ce is the application of the potential function for Hfinal, and gradpsi.ce its gradient.

Value

A numeric or a numeric vector.

Author(s)

Christophe Dutang

References

S. Bellavia, M. Macconi, B. Morini (2003), *An affine scaling trust-region approach to bound-constrained nonlinear systems*, Applied Numerical Mathematics 44, 257-280

A. Dreves, F. Facchinei, C. Kanzow and S. Sagratella (2011), *On the solutions of the KKT conditions of generalized Nash equilibrium problems*, SIAM Journal on Optimization 21(3), 1082-1108.

See Also

See also GNE.ceq.

---

projector *Projection of a point on a set*

Description

Projection of a point z on the set defined by the constraints g(x) <= 0.

Usage

```
projector(z, g, jacg, bounds=c(0, 10), echo=FALSE, ...)
```

Arguments

z	The point to project.
g	The constraint function.
jacg	The jacobian of the constraint function.
bounds	bounds for the randomized initial iterate.
echo	a logical to plot traces.
...	further arguments to pass to g function.

Details

Find a point x in the set K which minimizes the Euclidean distance $||z - x||^2$, where the set K is $x, g(x) <= 0$. The Optimization is carried out by the `constrOptim.nl` function of the package alabama.

Value

A vector x.

Author(s)

Christophe Dutang

See Also

See also GNE.

Examples

```
# 1. the rectangle set
#

g <- function(x)
c(x - 3, 1 - x)

jacg <- function(x)
rbind(
diag( rep(1, length(x)) ),
diag( rep(-1, length(x)) )
)

z <- runif(2, 3, 4)

#computation
projz <- projector(z, g, jacg)

#plot
plot(c(1, 3), c(1, 1), xlim=c(0, 4), ylim=c(0,4), type="l", col="blue")
lines(c(3, 3), c(1, 3), col="blue")
lines(c(3, 1), c(3, 3), col="blue")
lines(c(1, 1), c(3, 1), col="blue")

points(z[1], z[2], col="red")
points(projz[1], projz[2], col="red", pch="+")

z <- runif(2) + c(1, 0)
projz <- projector(z, g, jacg)

points(z[1], z[2], col="green")
points(projz[1], projz[2], col="green", pch="+")
```

```
# 2. the circle set
#

g <- function(x) sum((x-2)^2)-1
jacg <- function(x) as.matrix( 2*(x-2) )

z <- runif(2) + c(1, 0)

#computation
projz <- projector(z, g, jacg)

#plot
plot(c(1, 3), c(1, 1), xlim=c(0, 4), ylim=c(0,4), type="n", col="blue")
symbols(2, 2, circles=1, fg="blue", add=TRUE, inches=FALSE)

points(z[1], z[2], col="red")
points(projz[1], projz[2], col="red", pch="+")

z <- c(runif(1, 3, 4), runif(1, 1, 2))
projz <- projector(z, g, jacg)

points(z[1], z[2], col="green")
points(projz[1], projz[2], col="green", pch="+")
```

`rejection` *Rejection method for random generation.*

Description

Generate random variate satisfying the constraint function by the Rejection algorithm.

Usage

```
rejection(constr, nvars, LB=0, UB=1, ..., echo=FALSE,
method=c("unif","norm", "normcap"), control=list())
```

Arguments

`constr`	Constraint function
`nvars`	Number of variables
`LB`	Lower bound
`UB`	Upper bound
`...`	further arguments to pass to `constr` function.
`echo`	a logical to plot traces.

method	the distribution to draw random variates, either "unif", "norm", "normcap".
control	a named list containing the mean and the standard deviation of the normal distribution used if method!="unif".

Details

Draw random variates x until all the components of constr(x) are negative. The distribution to draw random variates can be the uniform distribution on the hypercube defined by LB and UB, the normal distribution centered in (LB + UB)/2 and standard deviation (UB - LB) / (4*1.9600) and the capped normal distribution (intended for debug use).

Value

A vector x which verifies the constraints constr(x) <= 0.

Author(s)

Christophe Dutang

See Also

See also GNE.

Examples

```
f <- function(x) x[1]^2 + x[2]^2 - 1

rejection(f, 2, -3, 3, method="unif")

rejection(f, 2, -3, 3, method="norm")
```

SSR *SemiSmooth Reformulation*

Description

functions of the SemiSmooth Reformulation of the GNEP

Usage

```
funSSR(z, dimx, dimlam, grobj, arggrobj, constr, argconstr, grconstr, arggrconstr,
compl, argcompl, dimmu, joint, argjoint, grjoint, arggrjoint, echo=FALSE)
jacSSR(z, dimx, dimlam, heobj, argheobj, constr, argconstr, grconstr, arggrconstr,
heconstr, argheconstr, gcompla, gcomplb, argcompl, dimmu, joint, argjoint,
grjoint, arggrjoint, hejoint, arghejoint, echo=FALSE)
```

Arguments

z	a numeric vector z containing $(x, lambda, mu)$ values.
dimx	a vector of dimension for x.
dimlam	a vector of dimension for $lambda$.
grobj	gradient of the objective function, see details.
arggrobj	a list of additional arguments of the objective gradient.
constr	constraint function, see details.
argconstr	a list of additional arguments of the constraint function.
grconstr	gradient of the constraint function, see details.
arggrconstr	a list of additional arguments of the constraint gradient.
compl	the complementarity function with (at least) two arguments: compl(a,b).
argcompl	list of possible additional arguments for compl.
dimmu	a vector of dimension for mu.
joint	joint function, see details.
argjoint	a list of additional arguments of the joint function.
grjoint	gradient of the joint function, see details.
arggrjoint	a list of additional arguments of the joint gradient.
heobj	Hessian of the objective function, see details.
argheobj	a list of additional arguments of the objective Hessian.
heconstr	Hessian of the constraint function, see details.
argheconstr	a list of additional arguments of the constraint Hessian.
gcompla	derivative of the complementarity function w.r.t. the first argument.
gcomplb	derivative of the complementarity function w.r.t. the second argument.
hejoint	Hessian of the joint function, see details.
arghejoint	a list of additional arguments of the joint Hessian.
echo	a logical to show some traces.

Details

Compute the SemiSmooth Reformulation of the GNEP: the Generalized Nash equilibrium problem is defined by objective functions Obj with player variables x defined in dimx and may have player-dependent constraint functions g of dimension dimlam and/or a common shared joint function h of dimension dimmu, where the Lagrange multiplier are $lambda$ and mu, respectively, see F. Facchinei et al.(2009) where there is no joint function.

Arguments of the Phi function The arguments which are functions must respect the following features

grobj The gradient $GradObj$ of an objective function Obj (to be minimized) must have 3 arguments for $GradObj(z, playnum, ideriv)$: vector z, player number, derivative index , and optionnally additional arguments in arggrobj.

`constr` The constraint function g must have 2 arguments: vector `z`, player number, such that $g(z, playnum) <= 0$. Optionnally, g may have additional arguments in `argconstr`.

`grconstr` The gradient of the constraint function g must have 3 arguments: vector `z`, player number, derivative index, and optionnally additional arguments in `arggrconstr`.

`compl` It must have two arguments and optionnally additional arguments in `argcompl`. A typical example is the minimum function.

`joint` The constraint function h must have 1 argument: vector `z`, such that $h(z) <= 0$. Optionnally, h may have additional arguments in `argjoint`.

`grjoint` The gradient of the constraint function h must have 2 arguments: vector `z`, derivative index, and optionnally additional arguments in `arggrjoint`.

Arguments of the Jacobian of Phi The arguments which are functions must respect the following features

`heobj` It must have 4 arguments: vector `z`, player number, two derivative indexes and optionnally additional arguments in `argheobj`.

`heconstr` It must have 4 arguments: vector `z`, player number, two derivative indexes and optionnally additional arguments in `argheconstr`.

`gcompla`,`gcomplb` It must have two arguments and optionnally additional arguments in `argcompl`.

`hejoint` It must have 3 arguments: vector `z`, two derivative indexes and optionnally additional arguments in `arghejoint`.

See the example below.

Value

A vector for `funSSR` or a matrix for `jacSSR`.

Author(s)

Christophe Dutang

References

F. Facchinei, A. Fischer and V. Piccialli (2009), *Generalized Nash equilibrium problems and Newton methods*, Math. Program.

See Also

See also `GNE.nseq`.

Examples

```
# (1) associated objective functions
#

dimx <- c(2, 2, 3)

#Gr_x_j O_i(x)
grfullob <- function(x, i, j)
{
```

```
x <- x[1:7]
if(i == 1)
{
grad <- 3*(x - 1:7)^2
}
if(i == 2)
{
grad <- 1:7*(x - 1:7)^(0:6)
}
if(i == 3)
{
s <- x[5]^2 + x[6]^2 + x[7]^2 - 5
grad <- c(1, 0, 1, 0, 4*x[5]*s, 4*x[6]*s, 4*x[7]*s)

}
grad[j]
}


#Gr_x_k Gr_x_j O_i(x)
hefullob <- function(x, i, j, k)
{
x <- x[1:7]
if(i == 1)
{
he <- diag( 6*(x - 1:7) )
}
if(i == 2)
{
he <- diag( c(0, 2, 6, 12, 20, 30, 42)*(x - 1:7)^c(0, 0:5) )
}
if(i == 3)
{
s <- x[5]^2 + x[6]^2 + x[7]^2

he <- rbind(rep(0, 7), rep(0, 7), rep(0, 7), rep(0, 7),
c(0, 0, 0, 0, 4*s+8*x[5]^2, 8*x[5]*x[6], 8*x[5]*x[7]),
c(0, 0, 0, 0, 8*x[5]*x[6], 4*s+8*x[6]^2, 8*x[6]*x[7]),
c(0, 0, 0, 0,  8*x[5]*x[7], 8*x[6]*x[7], 4*s+8*x[7]^2) )
}
he[j,k]
}



# (2) constraint linked functions
#

dimlam <- c(1, 2, 2)

#constraint function g_i(x)
g <- function(x, i)
{
```

```
x <- x[1:7]
if(i == 1)
res <- sum( x^(1:7) ) -7
if(i == 2)
res <- c(sum(x) + prod(x) - 14, 20 - sum(x))
if(i == 3)
res <- c(sum(x^2) + 1, 100 - sum(x))
res
}


#Gr_x_j g_i(x)
grfullg <- function(x, i, j)
{
x <- x[1:7]
if(i == 1)
{
grad <- (1:7) * x ^ (0:6)
}
if(i == 2)
{
grad <- 1 + sapply(1:7, function(i) prod(x[-i]))
grad <- cbind(grad, -1)
}
if(i == 3)
{
grad <- cbind(2*x, -1)
}


if(i == 1)
res <- grad[j]
if(i != 1)
res <- grad[j,]
as.numeric(res)
}



#Gr_x_k Gr_x_j g_i(x)
hefullg <- function(x, i, j, k)
{
x <- x[1:7]
if(i == 1)
{
he1 <- diag( c(0, 2, 6, 12, 20, 30, 42) * x ^ c(0, 0, 1:5) )
}
if(i == 2)
{
he1 <- matrix(0, 7, 7)
he1[1, -1] <- sapply(2:7, function(i) prod(x[-c(1, i)]))
he1[2, -2] <- sapply(c(1, 3:7), function(i) prod(x[-c(2, i)]))
he1[3, -3] <- sapply(c(1:2, 4:7), function(i) prod(x[-c(3, i)]))
```

```
he1[4, -4] <- sapply(c(1:3, 5:7), function(i) prod(x[-c(4, i)]))
he1[5, -5] <- sapply(c(1:4, 6:7), function(i) prod(x[-c(5, i)]))
he1[6, -6] <- sapply(c(1:5, 7:7), function(i) prod(x[-c(6, i)]))
he1[7, -7] <- sapply(1:6, function(i) prod(x[-c(7, i)]))


he2 <- matrix(0, 7, 7)

}
if(i == 3)
{
he1 <- diag(rep(2, 7))
he2 <- matrix(0, 7, 7)
}
if(i != 1)
return( c(he1[j, k], he2[j, k]) )
else
return( he1[j, k] )
}


# (3) compute Phi
#

z <- rexp(sum(dimx) + sum(dimlam))

n <- sum(dimx)
m <- sum(dimlam)
x <- z[1:n]
lam <- z[(n+1):(n+m)]

resphi <- funSSR(z, dimx, dimlam, grobj=grfullob, constr=g, grconstr=grfullg, compl=phiFB)


check <- c(grfullob(x, 1, 1) + lam[1] * grfullg(x, 1, 1),
grfullob(x, 1, 2) + lam[1] * grfullg(x, 1, 2),
grfullob(x, 2, 3) + lam[2:3] %*% grfullg(x, 2, 3),
grfullob(x, 2, 4) + lam[2:3] %*% grfullg(x, 2, 4),
grfullob(x, 3, 5) + lam[4:5] %*% grfullg(x, 3, 5),
grfullob(x, 3, 6) + lam[4:5] %*% grfullg(x, 3, 6),
grfullob(x, 3, 7) + lam[4:5] %*% grfullg(x, 3, 7),
phiFB( -g(x, 1), lam[1]),
phiFB( -g(x, 2)[1], lam[2]),
phiFB( -g(x, 2)[2], lam[3]),
phiFB( -g(x, 3)[1], lam[4]),
phiFB( -g(x, 3)[2], lam[5]))



#check
cat("\n\n_____________________________________\n\n")

#part A
```

```
print(cbind(check, res=as.numeric(resphi))[1:n, ])
#part B
print(cbind(check, res=as.numeric(resphi))[(n+1):(n+m), ])

# (4) compute Jac Phi
#

resjacphi <- jacSSR(z, dimx, dimlam, heobj=hefullob, constr=g, grconstr=grfullg,
heconstr=hefullg, gcompla=GrAphiFB, gcomplb=GrBphiFB)


#check
cat("\n\n_____________________________________\n\n")


cat("\n\npart A\n\n")


checkA <-
rbind(
c(hefullob(x, 1, 1, 1) + lam[1]*hefullg(x, 1, 1, 1),
hefullob(x, 1, 1, 2) + lam[1]*hefullg(x, 1, 1, 2),
hefullob(x, 1, 1, 3) + lam[1]*hefullg(x, 1, 1, 3),
hefullob(x, 1, 1, 4) + lam[1]*hefullg(x, 1, 1, 4),
hefullob(x, 1, 1, 5) + lam[1]*hefullg(x, 1, 1, 5),
hefullob(x, 1, 1, 6) + lam[1]*hefullg(x, 1, 1, 6),
hefullob(x, 1, 1, 7) + lam[1]*hefullg(x, 1, 1, 7)
),
c(hefullob(x, 1, 2, 1) + lam[1]*hefullg(x, 1, 2, 1),
hefullob(x, 1, 2, 2) + lam[1]*hefullg(x, 1, 2, 2),
hefullob(x, 1, 2, 3) + lam[1]*hefullg(x, 1, 2, 3),
hefullob(x, 1, 2, 4) + lam[1]*hefullg(x, 1, 2, 4),
hefullob(x, 1, 2, 5) + lam[1]*hefullg(x, 1, 2, 5),
hefullob(x, 1, 2, 6) + lam[1]*hefullg(x, 1, 2, 6),
hefullob(x, 1, 2, 7) + lam[1]*hefullg(x, 1, 2, 7)
),
c(hefullob(x, 2, 3, 1) + lam[2:3] %*% hefullg(x, 2, 3, 1),
hefullob(x, 2, 3, 2) + lam[2:3] %*% hefullg(x, 2, 3, 2),
hefullob(x, 2, 3, 3) + lam[2:3] %*% hefullg(x, 2, 3, 3),
hefullob(x, 2, 3, 4) + lam[2:3] %*% hefullg(x, 2, 3, 4),
hefullob(x, 2, 3, 5) + lam[2:3] %*% hefullg(x, 2, 3, 5),
hefullob(x, 2, 3, 6) + lam[2:3] %*% hefullg(x, 2, 3, 6),
hefullob(x, 2, 3, 7) + lam[2:3] %*% hefullg(x, 2, 3, 7)
),
c(hefullob(x, 2, 4, 1) + lam[2:3] %*% hefullg(x, 2, 4, 1),
hefullob(x, 2, 4, 2) + lam[2:3] %*% hefullg(x, 2, 4, 2),
hefullob(x, 2, 4, 3) + lam[2:3] %*% hefullg(x, 2, 4, 3),
hefullob(x, 2, 4, 4) + lam[2:3] %*% hefullg(x, 2, 4, 4),
hefullob(x, 2, 4, 5) + lam[2:3] %*% hefullg(x, 2, 4, 5),
hefullob(x, 2, 4, 6) + lam[2:3] %*% hefullg(x, 2, 4, 6),
hefullob(x, 2, 4, 7) + lam[2:3] %*% hefullg(x, 2, 4, 7)
),
c(hefullob(x, 3, 5, 1) + lam[4:5] %*% hefullg(x, 3, 5, 1),
```

```
hefullob(x, 3, 5, 2) + lam[4:5] %*% hefullg(x, 3, 5, 2),
hefullob(x, 3, 5, 3) + lam[4:5] %*% hefullg(x, 3, 5, 3),
hefullob(x, 3, 5, 4) + lam[4:5] %*% hefullg(x, 3, 5, 4),
hefullob(x, 3, 5, 5) + lam[4:5] %*% hefullg(x, 3, 5, 5),
hefullob(x, 3, 5, 6) + lam[4:5] %*% hefullg(x, 3, 5, 6),
hefullob(x, 3, 5, 7) + lam[4:5] %*% hefullg(x, 3, 5, 7)
),
c(hefullob(x, 3, 6, 1) + lam[4:5] %*% hefullg(x, 3, 6, 1),
hefullob(x, 3, 6, 2) + lam[4:5] %*% hefullg(x, 3, 6, 2),
hefullob(x, 3, 6, 3) + lam[4:5] %*% hefullg(x, 3, 6, 3),
hefullob(x, 3, 6, 4) + lam[4:5] %*% hefullg(x, 3, 6, 4),
hefullob(x, 3, 6, 5) + lam[4:5] %*% hefullg(x, 3, 6, 5),
hefullob(x, 3, 6, 6) + lam[4:5] %*% hefullg(x, 3, 6, 6),
hefullob(x, 3, 6, 7) + lam[4:5] %*% hefullg(x, 3, 6, 7)
),
c(hefullob(x, 3, 7, 1) + lam[4:5] %*% hefullg(x, 3, 7, 1),
hefullob(x, 3, 7, 2) + lam[4:5] %*% hefullg(x, 3, 7, 2),
hefullob(x, 3, 7, 3) + lam[4:5] %*% hefullg(x, 3, 7, 3),
hefullob(x, 3, 7, 4) + lam[4:5] %*% hefullg(x, 3, 7, 4),
hefullob(x, 3, 7, 5) + lam[4:5] %*% hefullg(x, 3, 7, 5),
hefullob(x, 3, 7, 6) + lam[4:5] %*% hefullg(x, 3, 7, 6),
hefullob(x, 3, 7, 7) + lam[4:5] %*% hefullg(x, 3, 7, 7)
)
)


print(resjacphi[1:n, 1:n] - checkA)


cat("\n\n_____________________________________\n\n")


cat("\n\npart B\n\n")


checkB <-
rbind(
cbind(c(grfullg(x, 1, 1), grfullg(x, 1, 2)), c(0, 0), c(0, 0), c(0, 0), c(0, 0)),
cbind(c(0, 0), rbind(grfullg(x, 2, 3), grfullg(x, 2, 4)), c(0, 0), c(0, 0)),
cbind(c(0, 0, 0), c(0, 0, 0), c(0, 0, 0),
 rbind(grfullg(x, 3, 5), grfullg(x, 3, 6), grfullg(x, 3, 7)))
)


print(resjacphi[1:n, (n+1):(n+m)] - checkB)


cat("\n\n_____________________________________\n\n")
cat("\n\npart C\n\n")


gx <- c(g(x,1), g(x,2), g(x,3))
```

```
checkC <-
- t(
cbind(
rbind(
grfullg(x, 1, 1) * GrAphiFB(-gx, lam)[1],
grfullg(x, 1, 2) * GrAphiFB(-gx, lam)[1],
grfullg(x, 1, 3) * GrAphiFB(-gx, lam)[1],
grfullg(x, 1, 4) * GrAphiFB(-gx, lam)[1],
grfullg(x, 1, 5) * GrAphiFB(-gx, lam)[1],
grfullg(x, 1, 6) * GrAphiFB(-gx, lam)[1],
grfullg(x, 1, 7) * GrAphiFB(-gx, lam)[1]
),
rbind(
grfullg(x, 2, 1) * GrAphiFB(-gx, lam)[2:3],
grfullg(x, 2, 2) * GrAphiFB(-gx, lam)[2:3],
grfullg(x, 2, 3) * GrAphiFB(-gx, lam)[2:3],
grfullg(x, 2, 4) * GrAphiFB(-gx, lam)[2:3],
grfullg(x, 2, 5) * GrAphiFB(-gx, lam)[2:3],
grfullg(x, 2, 6) * GrAphiFB(-gx, lam)[2:3],
grfullg(x, 2, 7) * GrAphiFB(-gx, lam)[2:3]
),
rbind(
grfullg(x, 3, 1) * GrAphiFB(-gx, lam)[4:5],
grfullg(x, 3, 2) * GrAphiFB(-gx, lam)[4:5],
grfullg(x, 3, 3) * GrAphiFB(-gx, lam)[4:5],
grfullg(x, 3, 4) * GrAphiFB(-gx, lam)[4:5],
grfullg(x, 3, 5) * GrAphiFB(-gx, lam)[4:5],
grfullg(x, 3, 6) * GrAphiFB(-gx, lam)[4:5],
grfullg(x, 3, 7) * GrAphiFB(-gx, lam)[4:5]
)
)
)



print(resjacphi[(n+1):(n+m), 1:n] - checkC)


cat("\n\n_____________________________________\n\n")

cat("\n\npart D\n\n")


checkD <- diag(GrBphiFB(-gx, lam))

print(resjacphi[(n+1):(n+m), (n+1):(n+m)] - checkD)
```

---

stepfunc *Step functions*

Description

Step functions for relaxation methods

Usage

```
purestep(k)
decrstep(k, param)
decrstep5(k)
decrstep10(k)
decrstep20(k)
```

Arguments

k	iteration number.
param	parameter for the decreasing step function after which the step decreases.

Details

The decrstep function is a decreasing step serie such that decrstep(k) equals to $1/2/(k-param)$ when $k > param$, 1/2, otherwise. Functions decrstep5, decrstep10, decrstep20 are just wrappers of decrstep.

The purestep function implements a constant step serie equaled to 1.

Value

A numeric.

Author(s)

Christophe Dutang

See Also

See also GNE and GNE.fpeq.

Examples

```
cbind(
purestep(1:20),
decrstep(1:20, 7),
decrstep5(1:20),
decrstep10(1:20),
decrstep20(1:20)
)
```

VIR *Nikaido Isoda Reformulation*

Description

functions of the Nikaido Isoda Reformulation of the GNEP

Usage

```
gapVIR(x, y, dimx, grobj, arggrobj, param=list(), echo=FALSE)
gradxgapVIR(x, y, dimx, grobj, arggrobj, heobj, argheobj, param=list(), echo=FALSE)
gradygapVIR(x, y, dimx, grobj, arggrobj, param=list(), echo=FALSE)
fpVIR(x, dimx, obj, argobj, joint, argjoint,
grobj, arggrobj, jacjoint, argjacjoint, param=list(),
echo=FALSE, control=list(), yinit=NULL, optim.method="default")
```

Arguments

x, y	a numeric vector.
dimx	a vector of dimension for x.
obj	objective function (to be minimized), see details.
argobj	a list of additional arguments.
grobj	gradient of the objective function, see details.
arggrobj	a list of additional arguments of the objective gradient.
heobj	Hessian of the objective function, see details.
argheobj	a list of additional arguments of the objective Hessian.
joint	joint function, see details.
argjoint	a list of additional arguments of the joint function.
jacjoint	gradient of the joint function, see details.
argjacjoint	a list of additional arguments of the joint Jacobian.
param	a list of parameters.
control	a list with control parameters for the fixed point algorithm.
yinit	initial point when computing the fixed-point function.
optim.method	optimization method when computing the fixed-point function.
echo	a logical to show some traces.

Details

gapVIR computes the Nikaido Isoda function of the GNEP, while gradxgapVIR and gradygapVIR give its gradient with respect to x and y. fpVIR computes the fixed-point function.

Value

A vector for `funSSR` or a matrix for `jacSSR`.

Author(s)

Christophe Dutang

References

A. von Heusinger & J. Kanzow (2009), *Optimization reformulations of the generalized Nash equilibrium problem using Nikaido-Isoda-type functions*, Comput Optim Appl .

F. Facchinei, A. Fischer and V. Piccialli (2009), *Generalized Nash equilibrium problems and Newton methods*, Math. Program.

See Also

See also `GNE.fpeq`.

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