

# Package ‘GUD’

July 1, 2024

**Title** Bayesian Modal Regression Based on the GUD Family

**Version** 1.0.2

**Description** Provides probability density functions and sampling algorithms for three key distributions from the General Unimodal Distribution (GUD) family: the Flexible Gumbel (FG) distribution, the Double Two-Piece (DTP) Student-t distribution, and the Two-Piece Scale (TPSC) Student-t distribution. Additionally, this package includes a function for Bayesian linear modal regression, leveraging these three distributions for model fitting. The details of the Bayesian modal regression model based on the GUD family can be found at Liu, Huang, and Bai (2024) <[doi:10.1016/j.csda.2024.108012](https://doi.org/10.1016/j.csda.2024.108012)>.

**URL** [https://github.com/rh8liuqy/Bayesian\\_modal\\_regression](https://github.com/rh8liuqy/Bayesian_modal_regression)

**License** GPL (>= 3)

**Encoding** UTF-8

**RoxygenNote** 7.3.1

**Biarch** true

**Depends** R (>= 3.4.0)

**Imports** methods, Rcpp (>= 1.0.12), RcppParallel (>= 5.0.1), rstan (>= 2.32.6), rstantools (>= 2.4.0), posterior (>= 1.5.0), Rdpack (>= 2.6)

**LinkingTo** BH (>= 1.66.0), Rcpp (>= 1.0.12), RcppEigen (>= 0.3.3.3.0), RcppParallel (>= 5.0.1), rstan (>= 2.32.6), StanHeaders (>= 2.32.0)

**Suggests** MASS, lattice, bayesplot (>= 1.11.1), loo (>= 2.7.0), knitr, rmarkdown

**RdMacros** Rdpack

**SystemRequirements** GNU make

**LazyData** true

**VignetteBuilder** knitr

**NeedsCompilation** yes

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**Repository** CRAN

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crime	<i>U.S. Statewide Crime data from the year 2003</i>
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## Description

This dataset is sourced from the 5th edition of "The Art and Science of Learning from Data" by Alan Agresti and Christine Franklin.

## Usage

crime

## Format

crime:

A data frame with 51 rows and 9 columns:

**state** The list of 50 states in the United States and the District of Columbia.

**violent crime rate** The annual number of murders, forcible rapes, robberies, and aggravated assaults per 100,000 people in the population.

**murder rate** The annual number of murders per 100,000 people in the population.

**poverty** Percentage of the residents with income below the poverty level.

**high school** Percentage of the adult residents who have at least a high school education.

**college** Percentage of the adult residents who have a college education.

**single parent** Percentage of families headed by a single parent.

**unemployed** Percentage of the adult residents who are unemployed.

**metropolitan** Percentage of the residents living in metropolitan areas.

## Source

[https://img1.wsimg.com/blobby/go/bbca5dba-4947-4587-b40a-db346c01b1b3/downloads/us\\_statewide\\_crime.csv?ver=1709965708861](https://img1.wsimg.com/blobby/go/bbca5dba-4947-4587-b40a-db346c01b1b3/downloads/us_statewide_crime.csv?ver=1709965708861)

dDTP

*The DTP-Student-t Distribution***Description**

The DTP-Student-t Distribution

**Usage**

```
dDTP(x, theta, sigma1, sigma2, delta1, delta2)
```

```
rDTP(n, theta, sigma1, sigma2, delta1, delta2)
```

**Arguments**

x	vector of quantiles.
theta	vector of the location parameters.
sigma1	vector of the scale parameters of the left skewed part.
sigma2	vector of the scale parameters of the right skewed part.
delta1	the degree of freedom of the left skewed part.
delta2	the degree of freedom of the right skewed part.
n	number of observations.

**Details**

The DTP-Student-t distribution has the density

$$f_{\text{DTP}}(y \mid \theta, \sigma_1, \sigma_2, \delta_1, \delta_2) = w f_{\text{LT}}(y \mid \theta, \sigma_1, \delta_1) + (1 - w) f_{\text{RT}}(y \mid \theta, \sigma_2, \delta_2),$$

where

$$w = \frac{\sigma_1 f(0 \mid \delta_2)}{\sigma_1 f(0 \mid \delta_2) + \sigma_2 f(0 \mid \delta_1)},$$

$f(0 \mid \delta)$  represents

$f((y - \theta)/\sigma \mid \delta)$  evaluated at  $y = \theta$ ,

$$f_{\text{LT}}(y \mid \theta, \sigma, \delta) = \frac{2}{\sigma} f\left(\frac{y - \theta}{\sigma} \mid \delta\right) \mathbb{I}(y < \theta),$$

and

$$f_{\text{RT}}(y \mid \theta, \sigma, \delta) = \frac{2}{\sigma} f\left(\frac{y - \theta}{\sigma} \mid \delta\right) \mathbb{I}(y \geq \theta).$$

Additionally,  $f(y \mid \delta)$  represents the density function of the standardized Student-t distribution with the degree of freedom  $\delta$ .

**Value**

dDTP gives the density. rDTP generates random deviates.

## References

Liu Q, Huang X, Bai R (2024). “Bayesian Modal Regression Based on Mixture Distributions.” *Computational Statistics & Data Analysis*, 108012. doi:10.1016/j.csda.2024.108012.

## Examples

```
set.seed(100)
require(graphics)

# Random Number Generation
X <- rDTP(n = 1e5, theta = 5, sigma1 = 7, sigma2 = 3, delta1 = 5, delta2 = 6)

# Plot the histogram
hist(X, breaks = 100, freq = FALSE)

# The red dashed line should match the underlining histogram
points(x = seq(-100, 40, length.out = 1000),
       y = dDTP(x = seq(-100, 40, length.out = 1000),
               theta = 5, sigma1 = 7, sigma2 = 3, delta1 = 5, delta2 = 6),
       type = "l",
       col = "red",
       lwd = 3,
       lty = 2)
```

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dFG

*The Flexible Gumbel Distribution*


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## Description

The Flexible Gumbel Distribution

## Usage

```
dFG(x, w, loc, sigma1, sigma2)
```

```
rFG(n, w, loc, sigma1, sigma2)
```

## Arguments

x	vector of quantiles.
w	vector of weight parameters.
loc	vector of the location parameters.
sigma1	vector of the scale parameters of the left skewed part.
sigma2	vector of the scale parameters of the right skewed part.
n	number of observations.

## Details

The Gumbel distribution has the density

$$f_{\text{Gumbel}}(y | \theta, \sigma) = \frac{1}{\sigma} \exp \left\{ -\frac{y - \theta}{\sigma} - \exp \left( -\frac{y - \theta}{\sigma} \right) \right\},$$

where  $\theta \in \mathbb{R}$  is the mode as the location parameter,  $\sigma > 0$  is the scale parameter.

The flexible Gumbel distribution has the density

$$f_{\text{FG}}(y | w, \theta, \sigma_1, \sigma_2) = w f_{\text{Gumbel}}(-y | -\theta, \sigma_1) + (1 - w) f_{\text{Gumbel}}(y | \theta, \sigma_2).$$

where  $w \in [0, 1]$  is the weight parameter,  $\sigma_1 > 0$  is the scale parameter of the left skewed part and  $\sigma_2 > 0$  is the scale parameter of the right skewed part.

## Value

dFG gives the density. rFG generates random deviates.

## References

Liu Q, Huang X, Bai R (2024). “Bayesian Modal Regression Based on Mixture Distributions.” *Computational Statistics & Data Analysis*, 108012. doi:10.1016/j.csda.2024.108012.

## Examples

```
set.seed(100)
require(graphics)

# Random Number Generation
X <- rFG(n = 1e5, w = 0.3, loc = 0, sigma1 = 1, sigma2 = 2)

# Plot the histogram
hist(X, breaks = 100, freq = FALSE)

# The red dashed line should match the underlining histogram
points(x = seq(-10, 20, length.out = 1000),
       y = dFG(x = seq(-10, 20, length.out = 1000),
              w = 0.3, loc = 0, sigma1 = 1, sigma2 = 2),
       type = "l",
       col = "red",
       lwd = 3,
       lty = 2)
```

dTPSC

*The TPSC-Student-t Distribution***Description**

The TPSC-Student-t Distribution

**Usage**

dTPSC(x, w, theta, sigma, delta)

rTPSC(n, w, theta, sigma, delta)

**Arguments**

x                    vector of quantiles.  
w                    vector of weight parameters.  
theta                vector of the location parameters.  
sigma                vector of the scale parameters.  
delta                the degree of freedom.  
n                    number of observations.

**Details**

The TPSC-Student-t distribution has the density

$$f_{\text{TPSC}}(y \mid w, \theta, \sigma, \delta) = w f_{\text{LT}}\left(y \mid \theta, \sigma \sqrt{\frac{w}{1-w}}, \delta\right) + (1-w) f_{\text{RT}}\left(y \mid \theta, \sigma \sqrt{\frac{1-w}{w}}, \delta\right),$$

where

$$f_{\text{LT}}(y \mid \theta, \sigma, \delta) = \frac{2}{\sigma} f\left(\frac{y-\theta}{\sigma} \mid \delta\right) \mathbb{I}(y < \theta),$$

and

$$f_{\text{RT}}(y \mid \theta, \sigma, \delta) = \frac{2}{\sigma} f\left(\frac{y-\theta}{\sigma} \mid \delta\right) \mathbb{I}(y \geq \theta).$$

Additionally,  $f(y \mid \delta)$  represents the density function of the standardized Student-t distribution with the degree of freedom  $\delta$ .

**Value**

dTPSC gives the density. rTPSC generates random deviates.

**References**

Liu Q, Huang X, Bai R (2024). “Bayesian Modal Regression Based on Mixture Distributions.” *Computational Statistics & Data Analysis*, 108012. doi:10.1016/j.csda.2024.108012.

### Examples

```
set.seed(100)
require(graphics)

# Random Number Generation
X <- rTPSC(n = 1e5, w = 0.7, theta = -1, sigma = 3, delta = 5)

# Plot the histogram
hist(X, breaks = 100, freq = FALSE)

# The red dashed line should match the underlining histogram
points(x = seq(-70, 50, length.out = 1000),
       y = dTPSC(x = seq(-70, 50, length.out = 1000),
                w = 0.7, theta = -1, sigma = 3, delta = 5),
       type = "l",
       col = "red",
       lwd = 3,
       lty = 2)
```

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GUD

*The 'GUD' package.*

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### Description

This R package encompasses the probability density functions of three key distributions: the flexible Gumbel distribution, the double two-piece Student-t distribution, and the two-piece scale Student-t distribution, all belonging to the general unimodal distribution family, along with their corresponding sampling algorithms. Additionally, the package offers a function for Bayesian linear modal regression, leveraging these three distributions for model fitting.

### Author(s)

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- Ray Bai <rbai@mailbox.sc.edu> ([ORCID](#))

### See Also

Useful links:

- [https://github.com/rh8liuqy/Bayesian\\_modal\\_regression](https://github.com/rh8liuqy/Bayesian_modal_regression)

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modal_regression	<i>Bayesian Modal Regression</i>
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## Description

Bayesian Modal Regression

## Usage

```
modal_regression(formula, data, model, ...)
```

## Arguments

formula	a formula.
data	a dataframe.
model	a description of the error distribution. Can be one of "FG", "DTP" and "TPSC".
...	Arguments passed to <code>rstan::sampling</code> (e.g. <code>iter</code> , <code>chains</code> ).

## Details

The Bayesian modal regression model based on the FG, DTP or TPSC distribution is defined as:

$$Y_i = \mathbf{X}_i\boldsymbol{\beta} + e_i,$$

where  $e_i$  follows the FG, DTP or TPSC distribution.

More details of the Bayesian modal regression model can be found at Liu, Huang, and Bai (2024) <https://arxiv.org/pdf/2211.10776>.

## Value

A draw object from the **posterior** package.

## References

Liu Q, Huang X, Bai R (2024). “Bayesian Modal Regression Based on Mixture Distributions.” *Computational Statistics & Data Analysis*, 108012. doi:10.1016/j.csda.2024.108012.

## Examples

```
# Save current user's options.
old <- options()
# (Optional - Running Multiple Chains in Parallel)
options(mc.cores = 2)

if (require(MASS)) { # Need Boston housing data from MASS package.
  # Fit the modal regression based on the FG distribution to the Boston housing data.
  FG_model <- modal_regression(formula = medv ~ .,
                              data = Boston,
```



```
        model = "FG",
        chains = 2,
        iter = 2000)
print(summary(FG_model), n = 17)

# Fit the modal regression based on the TPSC-Student-t distribution to the Boston housing data.
TPSC_model <- modal_regression(formula = medv ~ .,
                              data = Boston,
                              model = "TPSC",
                              chains = 2,
                              iter = 2000)
print(summary(TPSC_model), n = 17)

# Fit the modal regression based on the DTP-Student-t distribution to the Boston housing data.
DTP_model <- modal_regression(formula = medv ~ .,
                              data = Boston,
                              model = "DTP",
                              chains = 2,
                              iter = 2000)
print(summary(DTP_model), n = 17)
}

# reset (all) initial options
options(old)
```

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