Geometric Data Analysis with GDAtools

Nicolas Robette

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Contents

This tutorial presents the use of the `GDAtools` package for geometric data analysis. For further information on the statistical procedures themselves, it is recommended to refer to the books referenced on the home page of the website.

Introduction

For this example of Multiple Correspondence Analysis (MCA), we will use one of the data sets provided with the package. This is information about the tastes and cultural practices of 2000 individuals: listening to musical genres (French popular music, rap, rock, jazz and classical) and taste for film genres (comedy, crime film, animation, science fiction, love film, musical). These 11 variables will be used as "active" variables in the MCA and are completed by 3 "supplementary" variables: gender, age and education level.

```
library(ggplot2)  # for plots
library(GDAtools)
data(Taste)
str(Taste)
```

```
'data.frame':   2000 obs. of  14 variables:
 $ FrenchPop: Factor w/ 3 levels "No","Yes","NA": 2 1 2 1 2 1 1 1 1 2 ...
 $ Rap      : Factor w/ 3 levels "No","Yes","NA": 1 1 1 1 1 1 1 1 1 1 ...
 $ Rock     : Factor w/ 3 levels "No","Yes","NA": 1 1 2 1 1 2 1 1 2 1 ...
 $ Jazz     : Factor w/ 3 levels "No","Yes","NA": 1 2 1 1 1 1 1 1 1 1 ...
 $ Classical: Factor w/ 3 levels "No","Yes","NA": 1 2 1 2 1 1 1 1 1 1 ...
 $ Comedy   : Factor w/ 3 levels "No","Yes","NA": 1 2 1 1 1 1 2 2 2 2 ...
 $ Crime    : Factor w/ 3 levels "No","Yes","NA": 1 1 1 1 2 1 1 1 1 1 ...
 $ Animation: Factor w/ 3 levels "No","Yes","NA": 1 1 1 1 1 1 1 1 1 1 ...
 $ SciFi    : Factor w/ 3 levels "No","Yes","NA": 2 1 1 1 1 2 1 1 1 1 ...
 $ Love     : Factor w/ 3 levels "No","Yes","NA": 1 1 2 1 1 1 1 1 1 1 ...
 $ Musical  : Factor w/ 3 levels "No","Yes","NA": 1 1 1 1 1 1 1 1 1 1 ...
 $ Gender   : Factor w/ 2 levels "Men","Women": 1 1 2 1 2 2 2 2 1 1 ...
 $ Age      : Factor w/ 3 levels "15-24","25-49",..: 2 3 2 3 2 2 2 2 1 3 ...
 $ Educ     : Factor w/ 4 levels "None","Low","Medium",..: 3 4 3 4 2 1 3 2 2 2 ...
```

The active variables all have a "non response" ("NA") category, which concerns some individuals.

```
summary(Taste[,1:11])
```

```
 FrenchPop   Rap         Rock        Jazz        Classical  Comedy     Crime     
 No : 741   No :1730   No :1455   No :1621   No :1443   No :1141   No :1430  
 Yes:1249   Yes: 261   Yes: 535   Yes: 364   Yes: 552   Yes: 856   Yes: 555  
 NA :  10   NA :   9   NA :  10   NA :  15   NA :   5   NA :   3   NA :  15  
 Animation  SciFi       Love       Musical   
 No :1905   No :1845   No :1768   No :1923  
 Yes:  91   Yes: 143   Yes: 225   Yes:  66  
 NA :   4   NA :  12   NA :   7   NA :  11  
```

Specific MCA makes it possible to cancel these categories out in the construction of the factorial space, while keeping all the individuals. We start by locating the rank of the categories we want to cancel out.

```
getindexcat(Taste[,1:11])
```

```
 [1] "FrenchPop.No"  "FrenchPop.Yes" "FrenchPop.NA"  "Rap.No"       
 [5] "Rap.Yes"       "Rap.NA"        "Rock.No"       "Rock.Yes"     
 [9] "Rock.NA"       "Jazz.No"       "Jazz.Yes"      "Jazz.NA"      
[13] "Classical.No"  "Classical.Yes" "Classical.NA"  "Comedy.No"    
[17] "Comedy.Yes"    "Comedy.NA"     "Crime.No"      "Crime.Yes"    
[21] "Crime.NA"      "Animation.No"  "Animation.Yes" "Animation.NA" 
[25] "SciFi.No"      "SciFi.Yes"     "SciFi.NA"      "Love.No"      
[29] "Love.Yes"      "Love.NA"       "Musical.No"    "Musical.Yes"  
[33] "Musical.NA"   
```

The vector of these ranks is then given as an argument in the specific MCA function `speMCA()`.

```
mca <- speMCA(Taste[,1:11], excl=c(3,6,9,12,15,18,21,24,27,30,33))
```

Alternatively, we can use the list of *labels* of the "junk" categories.

```
junk <- c("FrenchPop.NA", "Rap.NA", "Rock.NA", "Jazz.NA", "Classical.NA",
          "Comedy.NA", "Crime.NA", "Animation.NA", "SciFi.NA", "Love.NA",
          "Musical.NA")
mca <- speMCA(Taste[,1:11], excl = junk)
```

The labels of these categories can be identified using the function `ijunk()`, which launches an interactive application for this purpose and allows to copy and paste the appropriate code.

Clouds

The Benzécri corrected inertia rates give an idea of how much information is represented by each axis.

```
modif.rate(mca)$modif
```

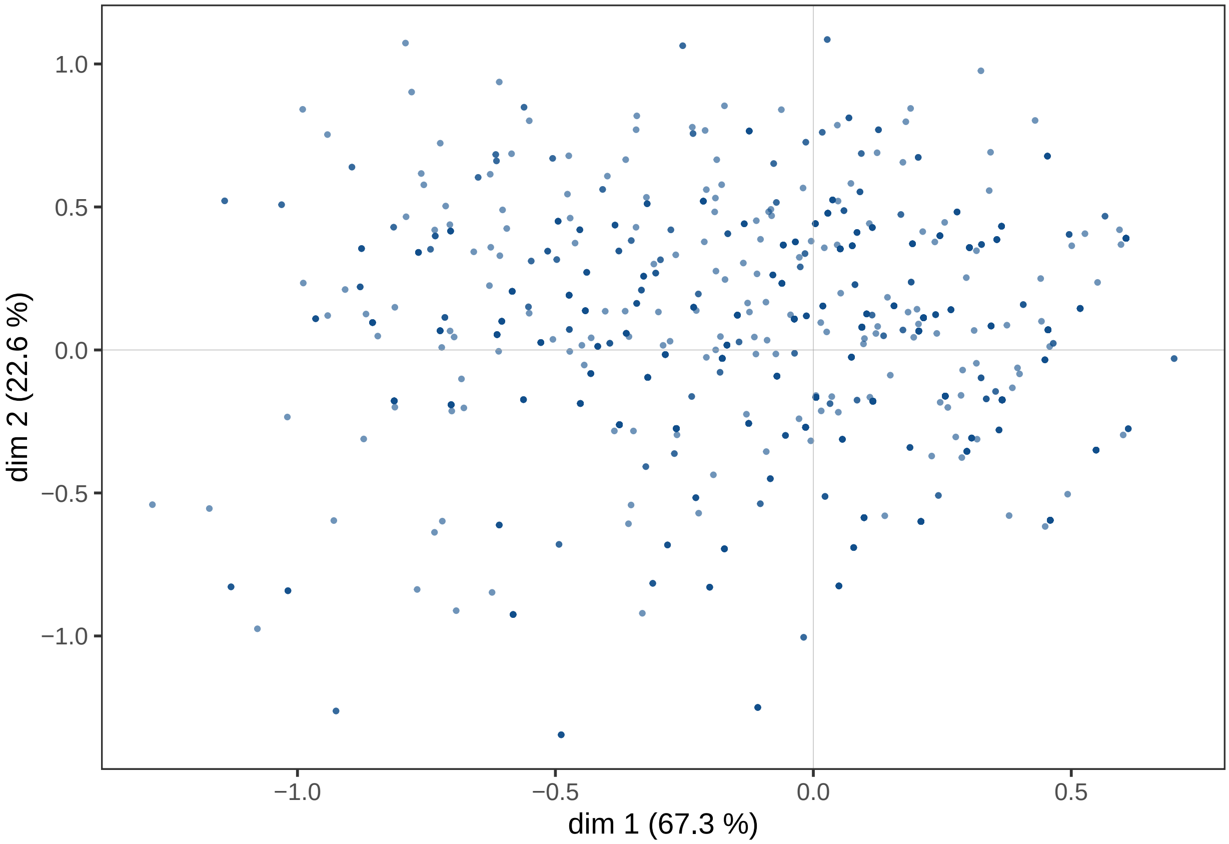
```
        mrate cum.mrate
1 67.30532896  67.30533
2 22.64536000  89.95069
3  7.17043134  97.12112
4  2.26387669  99.38500
5  0.59232858  99.97733
6  0.02267443 100.00000
```

We can see here that the first two axes capture most of the information (nearly 90%). In the following, we will therefore concentrate on the plane formed by axes 1 and 2.

Cloud of individuals

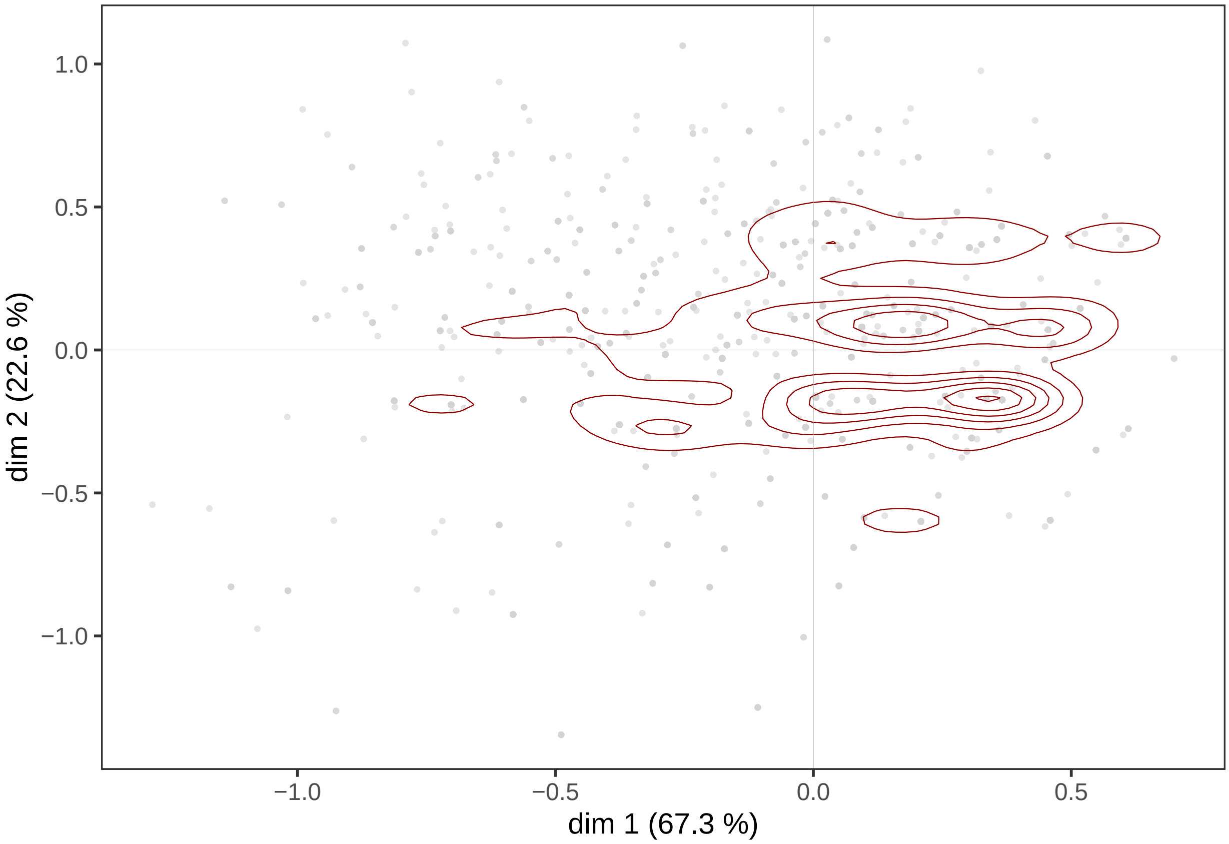
The cloud of individuals does not have a particular shape (triangle, horseshoe…), the points seem to be distributed in the whole plane (1,2).

```
ggcloud_indiv(mca)
```

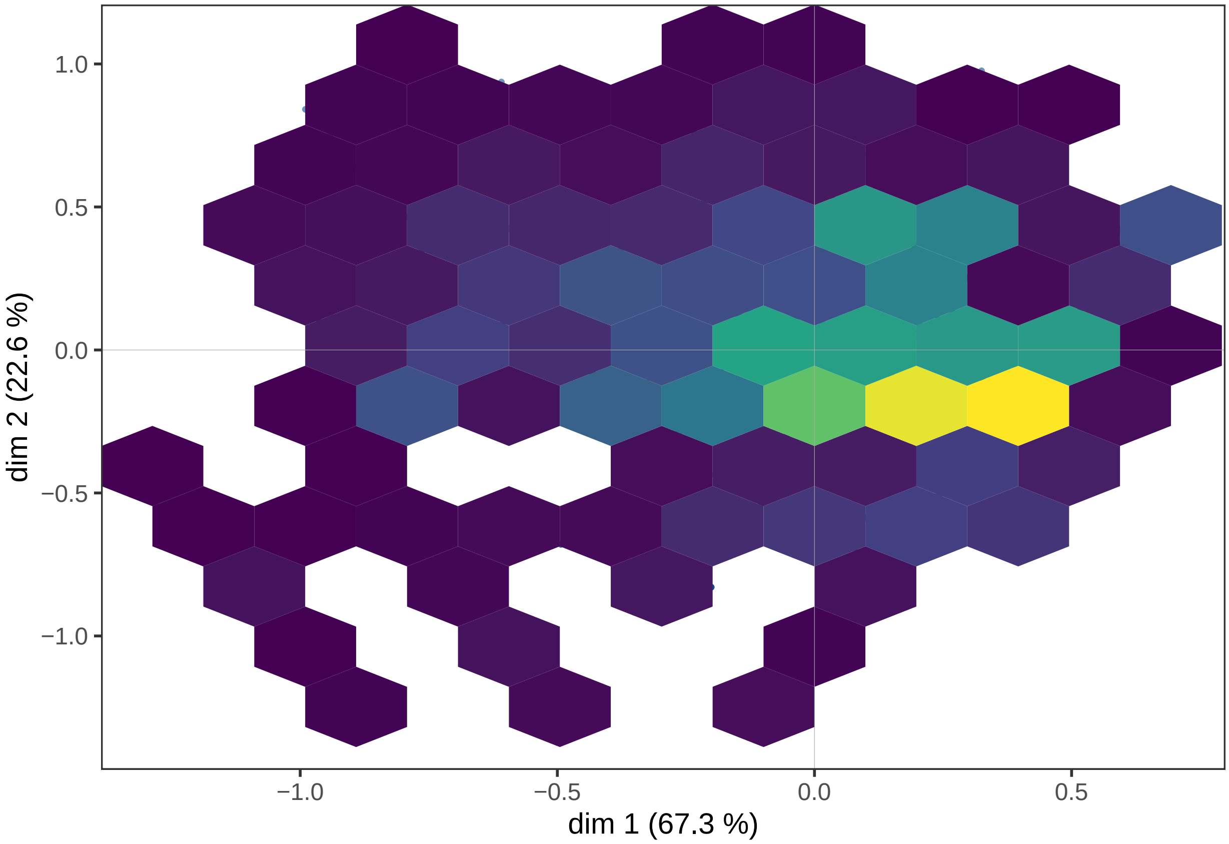


However, in some cases, points may overlap and the structure of the cloud of individuals is only imperfectly rendered by a scatter plot. It is then possible to complete the first graph by a representation of the density of points in the plane. The function `ggcloud_indiv()` allows this to be done using contours (like the contour lines of a topographic map) or hexagonal surfaces (colored with a color gradient according to the number of points located in the hexagon).

```
ggcloud_indiv(mca, col = "lightgray", density = "contour")
```



```
ggcloud_indiv(mca, density = "hex", hex.bin = 10)
```



Regardless of the density representation used, we observe that the points seem to be more concentrated in an area immediately to the right of the vertical axis.

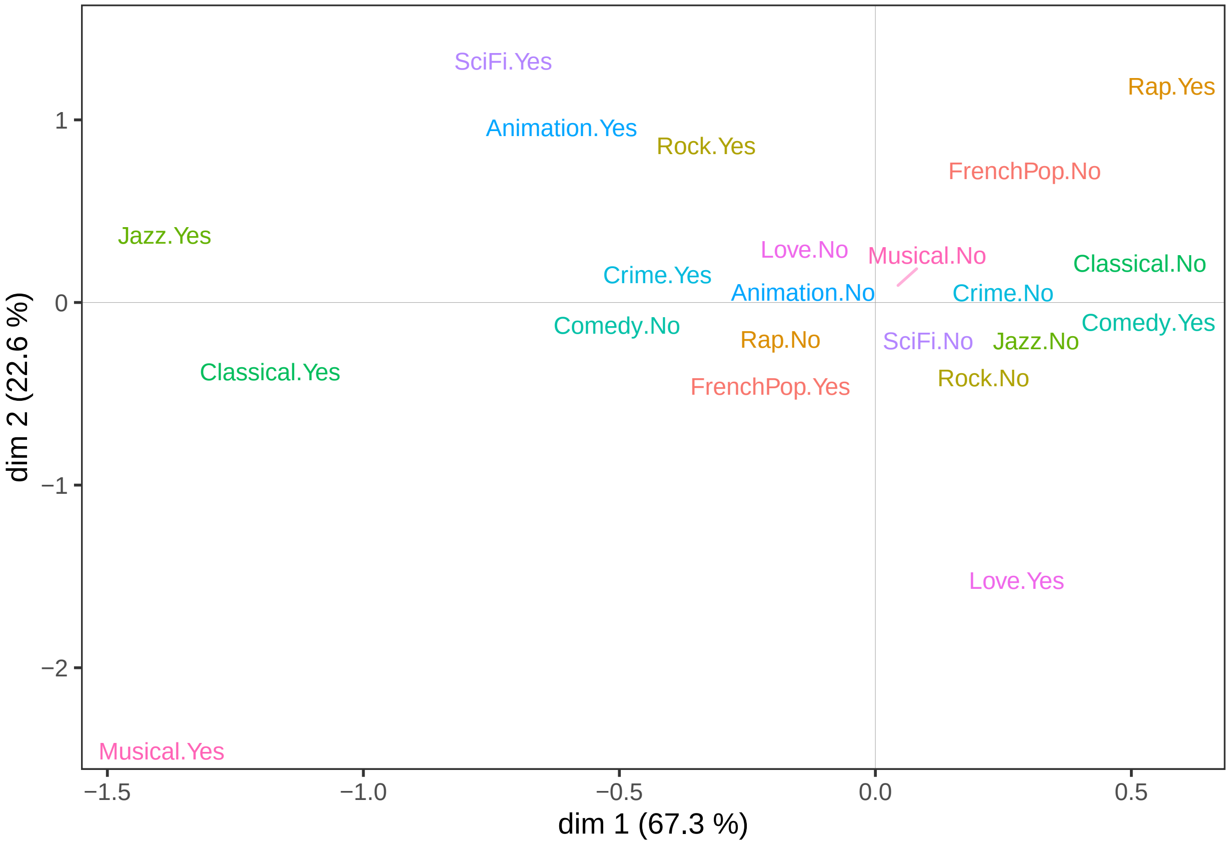
NB: `ggcloud_indiv()` also allows the *name* of the observations to be displayed, when this is of interest from an interpretation point of view.

Cloud of variables

On the cloud of variables:

- listening to jazz and classical music and liking musicals seem to be opposed to listening to rap and liking comedies on axis 1;
- the taste for animation and science fiction to the taste for love movies and musicals on axis 2.

```
ggcloud_variables(mca, shapes = FALSE, legend = "none")
```

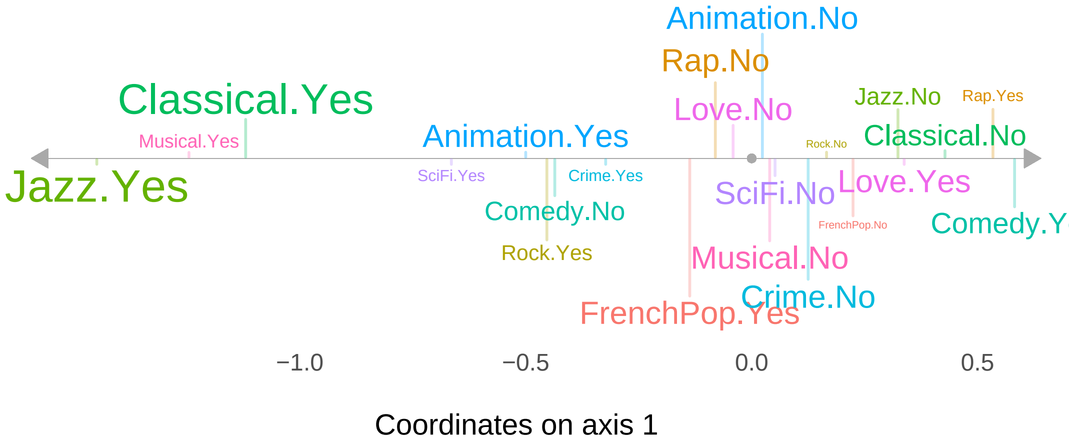


Many options are possible, including:

- adding symbols (circles, triangles, etc.) in addition to the labels of the categories ;
- selection or highlighting of the most important categories according to different criteria (contribution, quality of representation, typicality, etc.);
- using size, italics, bold and underlining to identify the most important categories.

The results are traditionally represented in a plane, i.e. in two dimensions (here dimensions 1 and 2). But it is sometimes easier to concentrate the interpretation on one axis at a time, which is possible with the function `ggaxis_variables()` (parameterized here to display the labels of the categories with a size proportional to their contribution to the construction of the axis).

```
ggaxis_variables(mca, axis = 1, prop = "ctr")
```



However, to be robust, the interpretation of the factorial plane cannot stop at a visual examination of the cloud of variables. This must be completed by a careful analysis of statistical indicators, in particular the **contributions** of the categories to the construction of the axes and their **quality of representation**.

Aids to interpretation

Most of the aids to interpretation and other useful information are present in the object created by `speMCA()` function. The package provides several functions to extract and organize this information.

- `contrib()` presents the contributions of the variables and the categories of these variables to the construction of each axis and to the construction of the cloud.
- `dimcontrib()` extracts the contributions of the individuals and the categories of variables to the construction of a particular axis.
- `dimdescr()` identifies the variables and variable categories most statistically associated with the different axes. The measures of association used are the **correlation ratios** (**eta²**) for the variables and the **correlation coefficients** for the categories.
- The function `planecontrib()` extracts the contributions of individuals and of the categories of variables to the construction of a particular *plane*.

The function `tabcontrib()` allows, for a given axis, to summarize the main contributions (by default, only the contributions above the average are presented).

```
tabcontrib(mca, dim = 1)
```

Variable	Category	Weight	Quality of representation	Contribution (left)	Contribution (right)	Total contribution	Cumulated contribution	Contribution of deviation	Proportion to variable
Classical	Yes	552	0.478	23.08		31.88	31.88	31.88	100
	No	1443	0.474		8.8				
Jazz	Yes	364	0.467	25.49		31.15	63.04	31.15	100
	No	1621	0.448		5.66				
Comedy	Yes	856	0.253		9.66	16.88	79.92	16.88	100
	No	1141	0.252	7.22					

The classical music and jazz listening variables alone contribute more than 60% to the construction of axis 1. Listening to classical music and jazz is thus opposed to not listening to them, and secondarily to the taste for comedies.

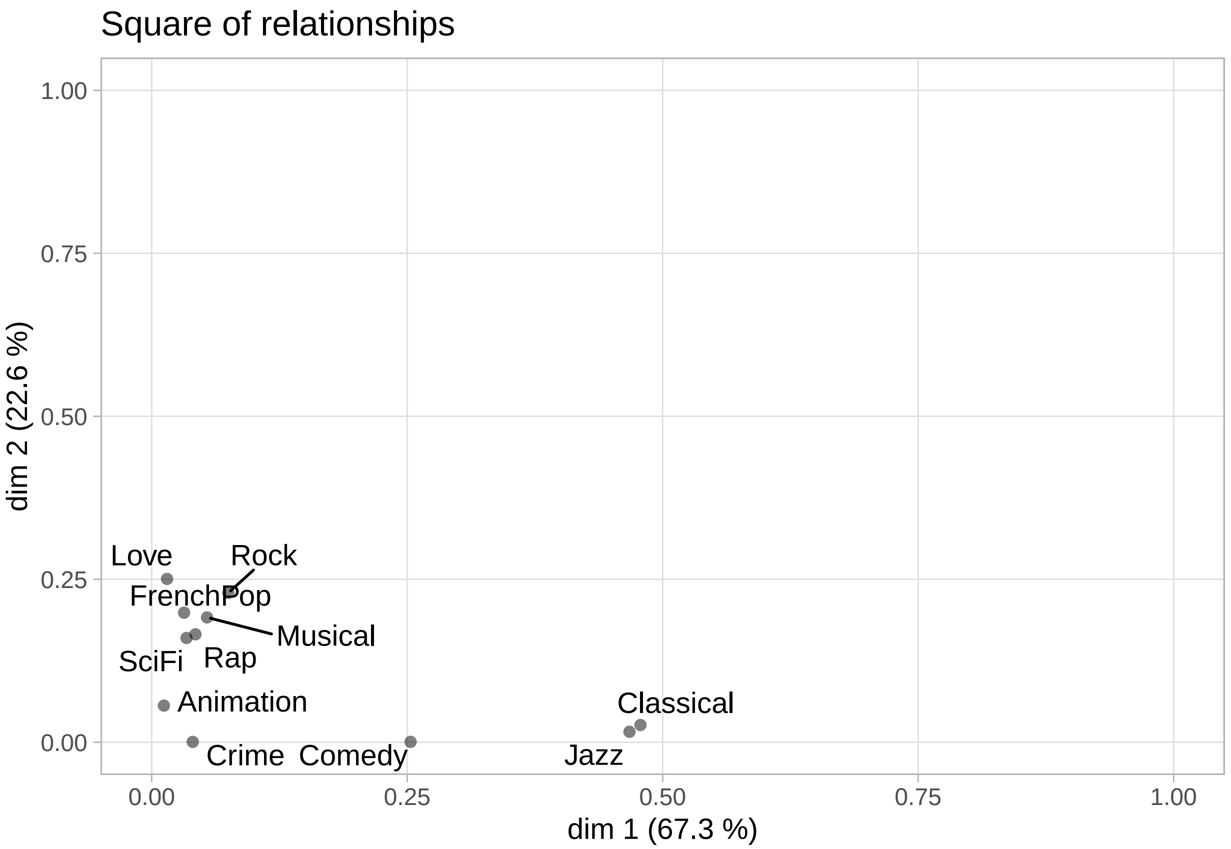
```
tabcontrib(mca, dim = 2)
```

Variable	Category	Weight	Quality of representation	Contribution (left)	Contribution (right)	Total contribution	Cumulated contribution	Contribution of deviation	Proportion to variable
Rock	Yes	535	0.229		12.99	17.77	17.77	17.77	100
	No	1455	0.226	4.78					
Love	Yes	225	0.250	17.24		17.24	35	17.24	88.79
FrenchPop	No	741	0.199		9.69	15.39	50.39	15.39	100
	Yes	1249	0.196	5.69					
Musical	Yes	66	0.191	14.33		14.33	64.72	14.33	96.61
SciFi	Yes	143	0.160		11.49	11.49	76.21	11.49	92.72
Rap	Yes	261	0.164		11.05	11.05	87.27	11.05	86.56

On axis 2, listening to rock and rap music and the taste for science fiction films are opposed to the taste for love films and musicals and the listening of French variety music.

One way to summarize the relationships between the active variables and the factorial axes is the so-called "square of relationships" (*carré des liaisons*) proposed by Escofier and Pagès (2008). It uses correlation ratios to represent these relationships.

```
ggeta2_variables(mca) + ggtitle("Square of relationships")
```



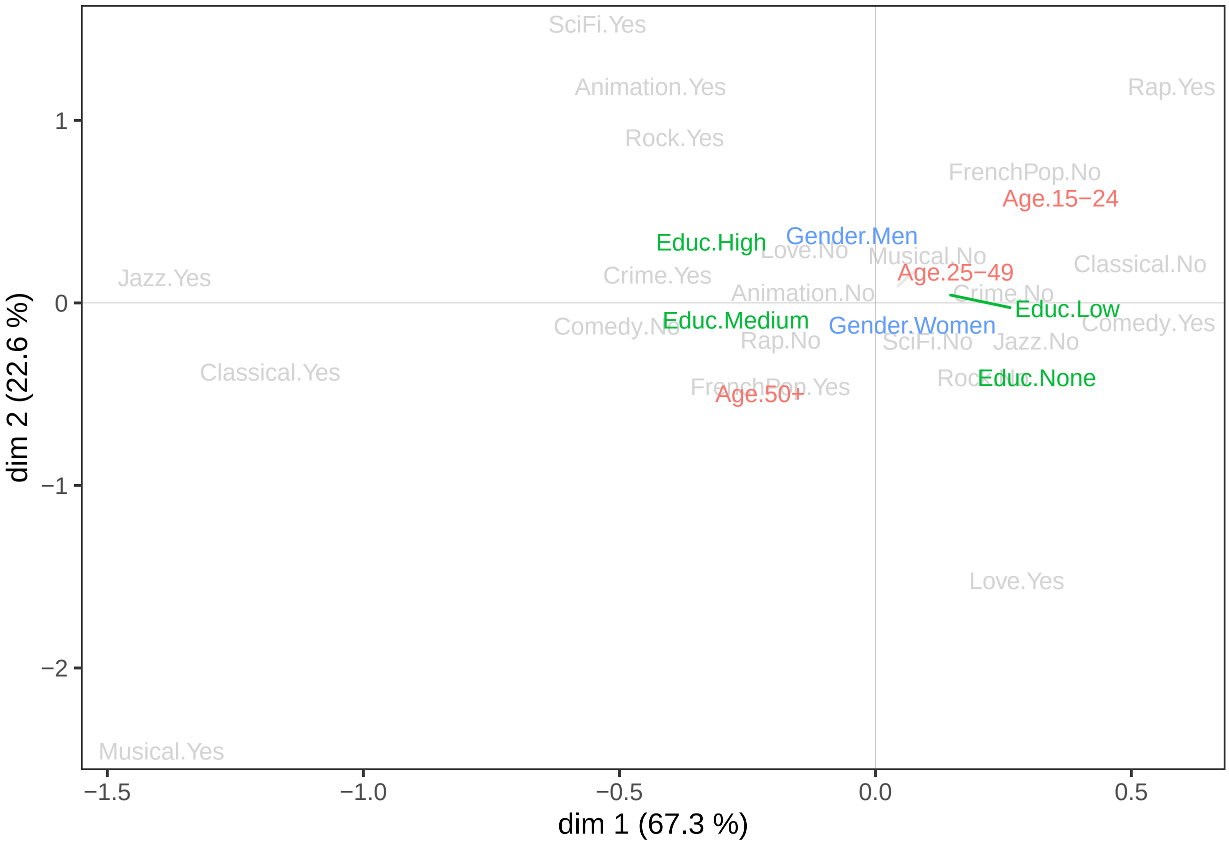
We see that listening to jazz and classical music is highly correlated with axis 1 and not at all with axis 2, and that the active variables most correlated with axis 2 are only moderately so.

Structuring factors

Supplementary variables

We can go further by studying the relationship between the factorial space and the supplementary variables, in this case gender, age and education. A first step is to project the supplementary variables onto the cloud of variables.

```
vcloud <- ggcloud_variables(mca, shapes=FALSE, col="lightgray")
ggadd_supvars(vcloud, mca, Taste[,c("Gender","Age","Educ")])
```



It is possible to set the display of each of the supplementary variables separately with the function `ggadd_supvar()` (without "s"). However, there are several drawbacks: more lines of code and the risk of overlapping of the labels of the categories.

The level of education seems to be associated above all with axis 1, the most educated being on the side of jazz and classical music listening. Gender seems to be linked only to axis 2, with women at the bottom of the plane and men at the top. As for age, it is associated with both axes: individuals move from the northeast quadrant to the southwest quadrant as their age increases.

These initial observations can be statistically confirmed by measuring the degree of association between the supplementary variables and the axes using the **correlation ratio** (**eta²**).

```
dimeta2(mca, Taste[,c("Gender","Age","Educ")])
```

```
       dim.1 dim.2
Gender   0.0   6.0
```

```
Age       3.8  14.2
Educ      5.9   3.3
```

Education is the supplementary variable most associated with axis 1: it “explains” 5.9% of the variance of individual coordinates on this axis. Age is also associated with the first axis, but to a lesser extent, and gender not at all.

On axis 2, age is the most structuring variable, ahead of gender and education level. We also see that age is clearly more related to axis 2 than to axis 1.

At the level of the categories, the association of an supplementary variable category with an axis can be characterized from the **correlation coefficients**.

```
des <- dimdescr(mca, vars = Taste[,c("Gender","Age","Educ")])
des$dim.1$categories
```

categories	mean.y.in.xcat	mean.y.global	sd.y.in.xcat	sd.y.global	cor
Age.15-24	0.156	0	0.355	0.369	0.176
Educ.None	0.094	0	0.324	0.369	0.145
Educ.Low	0.052	0	0.355	0.369	0.100
Age.25-49	0.006	0	0.358	0.369	0.015
Gender.Women	0.004	0	0.370	0.369	0.012
Gender.Men	-0.005	0	0.368	0.369	-0.012
Educ.Medium	-0.040	0	0.376	0.369	-0.051
Age.50+	-0.061	0	0.369	0.369	-0.142
Educ.High	-0.141	0	0.382	0.369	-0.213

On axis 1, those with no or little education and those aged 15-24 are opposed to those with more education and those aged 50 and over. The other categories appear to have little relationship with the axis (their correlation coefficients, in the last column of the table, are close to 0).

```
des$dim.2$categories
```

categories	mean.y.in.xcat	mean.y.global	sd.y.in.xcat	sd.y.global	cor
Age.15-24	0.236	0	0.320	0.342	0.288
Gender.Men	0.088	0	0.302	0.342	0.245
Age.25-49	0.049	0	0.333	0.342	0.123
Educ.High	0.076	0	0.327	0.342	0.123
Educ.Low	0.016	0	0.341	0.342	0.034
Educ.Medium	0.007	0	0.335	0.342	0.009
Educ.None	-0.100	0	0.341	0.342	-0.167
Gender.Women	-0.081	0	0.357	0.342	-0.245
Age.50+	-0.131	0	0.299	0.342	-0.330

On axis 2, men, those under 50, and those with higher education contrast with women, those over 50, and those with no degree.

Analysis of one supplementary variable

Let’s continue the analysis by focusing on *one* supplementary variable, education level. The function `supvar()` provides:

- the coordinates of the categories on the axes,
- their *squared cosines* (which gives the quality of representation of a category on an axis),

- their dispersion (variance) on the axes,
- the correlation ratios (eta^2) between the variable and the axes,
- the *typicality tests* (to which we will return in the next section),
- the *correlation coefficients* between the categories and the axes.

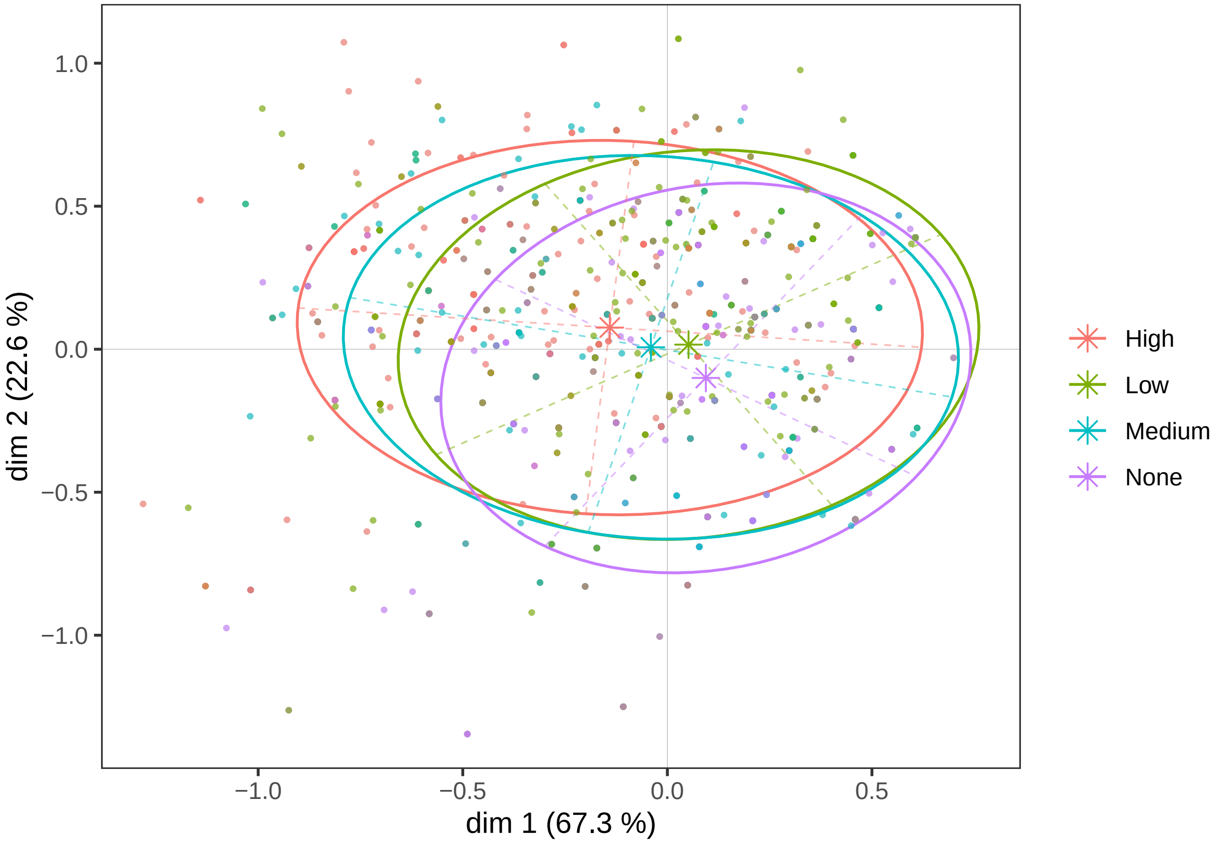
```
supvar(mca, Taste$Educ)
```

```
$weight
  None    Low Medium   High 
   490    678    357    475 

$coord
           dim.1     dim.2     dim.3     dim.4     dim.5
None    0.254716 -0.293253 -0.191563  0.094510 -0.050520
Low     0.140147  0.046951  0.002259 -0.063613 -0.044181
Medium -0.109057  0.019669  0.054436  0.070183 -0.027962
High   -0.380835  0.220714  0.153475 -0.059443  0.136194

$cos2
          dim.1    dim.2    dim.3    dim.4    dim.5
None   0.021054 0.027906 0.011908 0.002899 0.000828
Low    0.010073 0.001131 0.000003 0.002075 0.001001
Medium 0.002584 0.000084 0.000644 0.001070 0.000170
High   0.045175 0.015173 0.007337 0.001101 0.005777

$var
           dim.1    dim.2    dim.3    dim.4    dim.5
None    0.104915 0.116125 0.115232 0.103800 0.086031
Low     0.125921 0.116092 0.109029 0.087945 0.087735
Medium  0.141298 0.112505 0.097031 0.102449 0.094541
High    0.145983 0.107170 0.091066 0.106049 0.113352
within  0.128284 0.113341 0.104141 0.098718 0.094616
between 0.008061 0.003923 0.001598 0.000524 0.000555
total   0.136345 0.117264 0.105739 0.099242 0.095172
eta2    0.059123 0.033455 0.015115 0.005279 0.005832

$typic
           dim.1     dim.2     dim.3     dim.4     dim.5
None    6.487410 -7.468929 -4.878958  2.407110 -1.286704
Low     4.487336  1.503312  0.072336 -2.036827 -1.414633
Medium -2.272883  0.409932  1.134503  1.462690 -0.582753
High   -9.502876  5.507427  3.829615 -1.483270  3.398406

$pval
          dim.1    dim.2    dim.3    dim.4    dim.5
None   0.000000 0.000000 0.000001 0.016079 0.198197
Low    0.000007 0.132758 0.942334 0.041667 0.157176
Medium 0.023033 0.681856 0.256584 0.143552 0.560059
High   0.000000 0.000000 0.000128 0.138003 0.000678

$cor
        dim.1  dim.2  dim.3  dim.4  dim.5
None    0.145 -0.167 -0.109  0.054 -0.029
```

```
Low     0.100  0.034  0.002 -0.046 -0.032
Medium -0.051  0.009  0.025  0.033 -0.013
High   -0.213  0.123  0.086 -0.033  0.076
```

Graphically, we can represent the *subcloud* of each category using a **concentration ellipse**, which is centered on the mean point and encompasses 86% of individuals with that category.

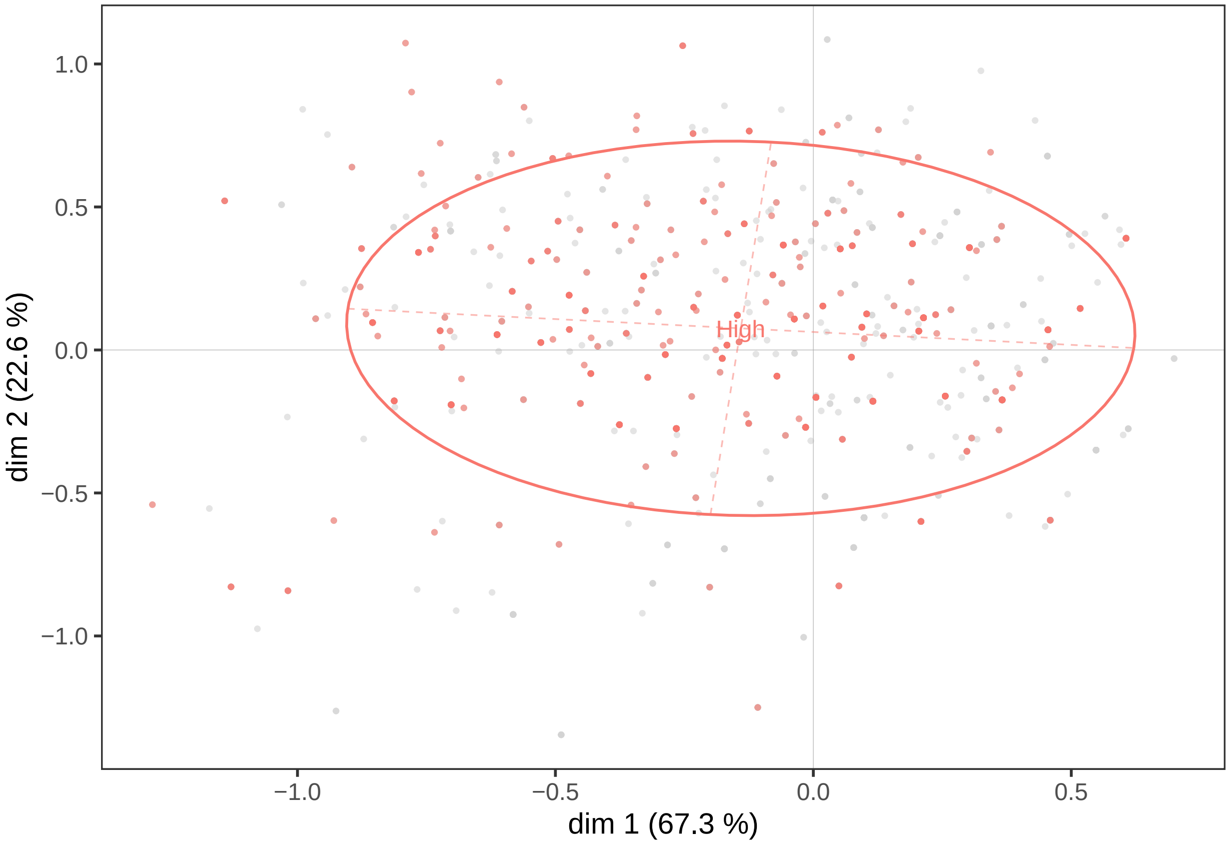
```
icloud <- ggcloud_indiv(mca, col = "lightgrey")
ggadd_kellipses(icloud, mca, Taste$Educ, label = FALSE)
```



Even if, as we have seen, the education level categories are ordered along axis 1, the subclouds overlap to a large extent, because the association between the variable and the axis is moderate.

Let us now look more specifically at the subcloud of the most highly educated individuals (the 4th category of the variable `Educ`).

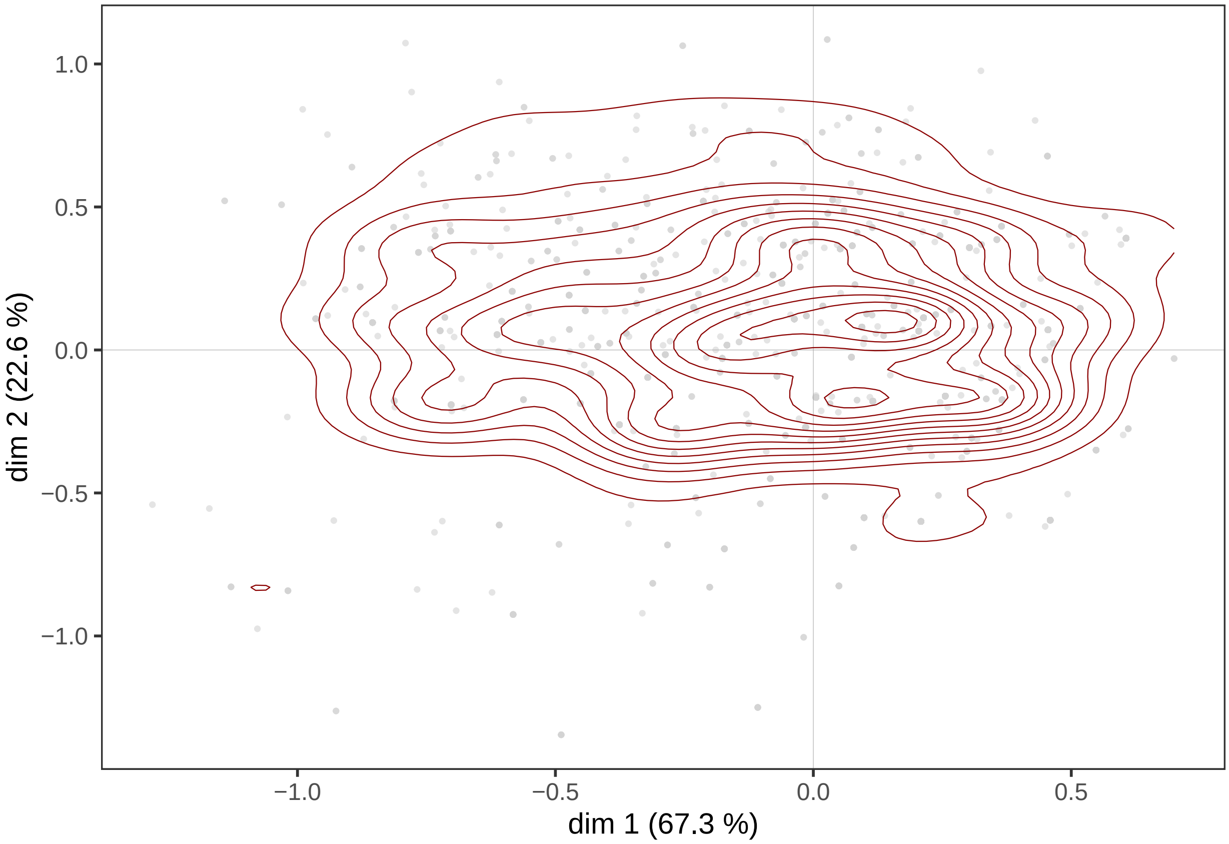
```
ggadd_kellipses(icloud, mca, Taste$Educ, sel = 4, legend = "none")
```



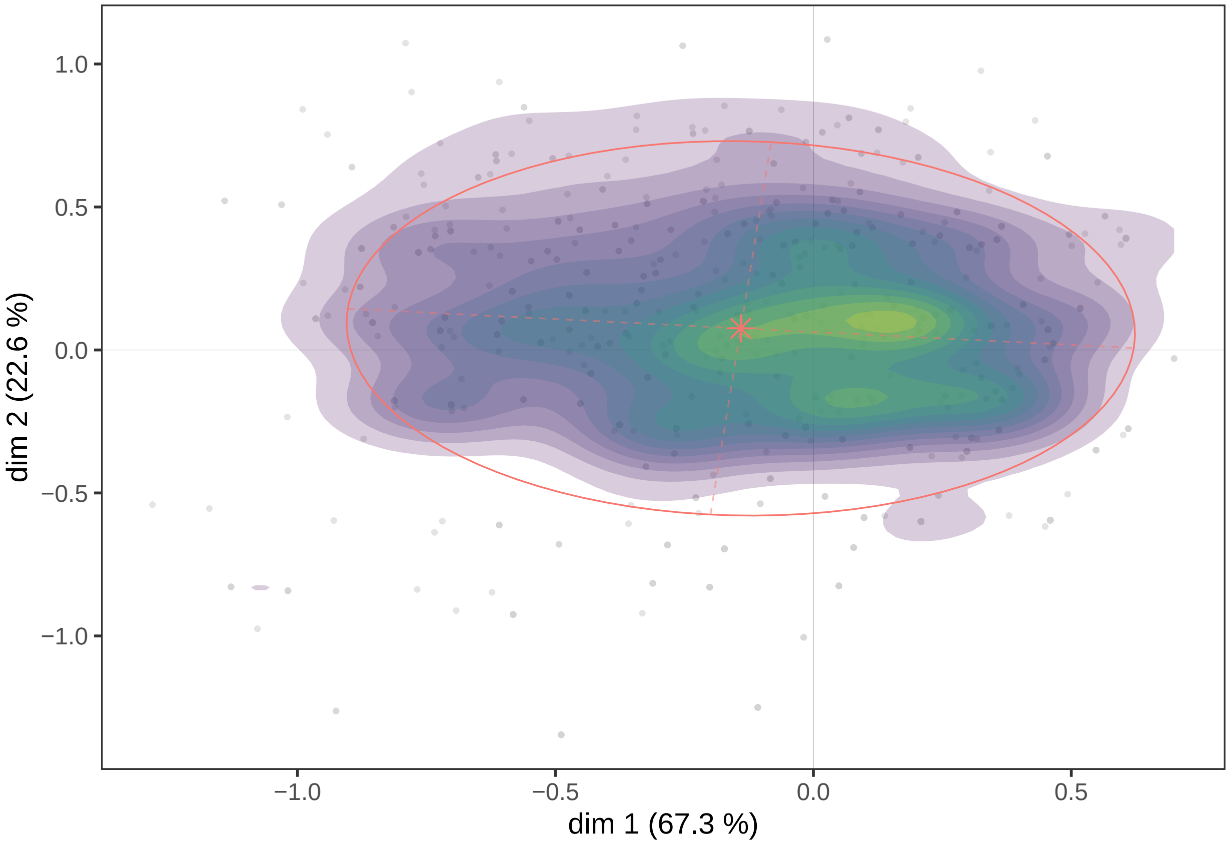
A concentration ellipse is useful because it jointly represents the mean point of the category and the dispersion of the subcloud along the axes. Here, we see that although the mean point of the most highly educated individuals is located in the northwest quadrant, a non-negligible portion of the points in the subcloud are to the right and/or bottom of the plane.

The concentration ellipses, on the other hand, give only an imperfect representation of the distribution of the points of the subcloud, because of the possible superposition of the points (in a similar way to what we have seen for the cloud of individuals) and its centering on the mean point. It can therefore be interesting to complete a concentration ellipse with a representation of the density of the points in the subcloud, in the form of contours or surfaces.

```
ggadd_density(icloud, mca, var = Taste$Educ, cat = "High", density = "contour")
```



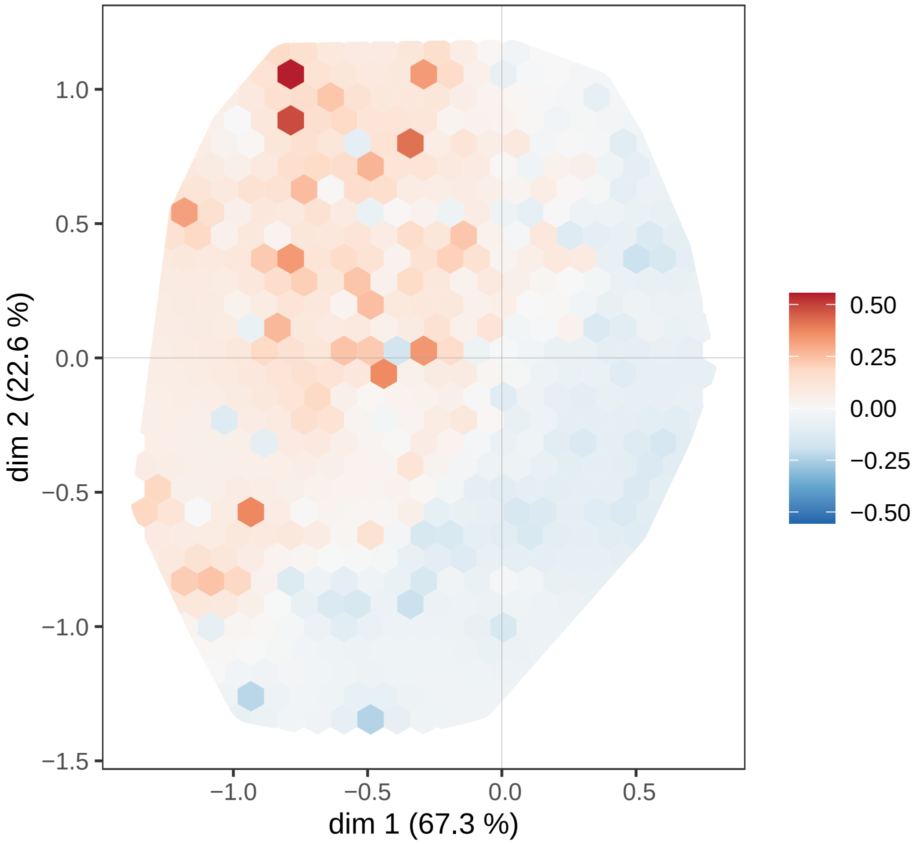
```
ggadd_density(icloud, mca, var = Taste$Educ, cat = "High", density = "area", ellipse = TRUE)
```



Here we see that there appears to be a concentration of highly educated individuals immediately to the right of the vertical axis, in an area that also corresponds to a concentration of points in the cloud of individuals (see above).

A further step in the analysis consists in neutralizing the influence of the distribution of the points of the cloud of individuals on that of the subcloud, by asking in which parts of the plane the most highly educated are over/under represented. In the following graph, we represent in each hexagonal tile the proportion of highly educated individuals, centered in relation to the proportion of highly educated individuals in the whole sample. In other words, we represent the over/under-representation of highly educated individuals in each portion of the plane. The graph is made more readable by a smoothing procedure derived from spatial analysis in geography (*inverse distance weighting*).

```
ggsmoothed_supvar(mca, var = Taste$Educ, cat = "High", center = TRUE)
```

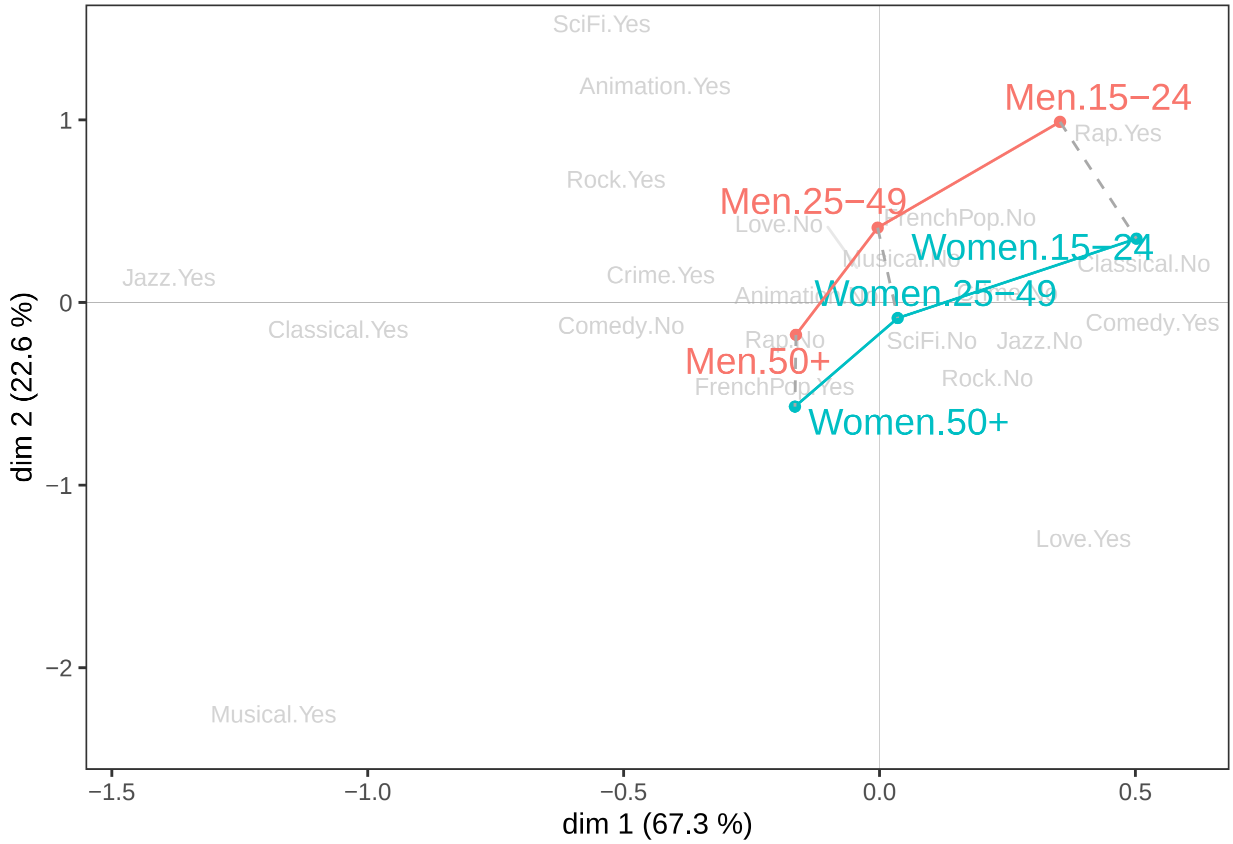


The most highly educated individuals are overrepresented throughout the northwest quadrant, but also among the westernmost of the southwest quadrant.

Interaction between two supplementary variables

Interactions between several supplementary variables can also be studied, for example here between gender and age.

```
ggadd_interaction(vcloud, mca, Taste$Gender, Taste$Age, legend = "none")
```



Gender and age appear to interact little in the plane (1,2). Note, however, that on axis 1, the differences between the youngest and oldest individuals are larger for women than for men, and that on axis 2, the youngest individuals differ more by gender than the oldest.

Note: The analyses performed here on supplementary variables are similarly feasible for supplementary *individuals*, using the functions `supind()` and `ggadd_supind()`. This is useful when some individuals were excluded from the MCA because they were too atypical or because some information was missing, for example.

Inductive analysis

If one wishes to assess the generalizability of the results, one can complement the descriptive analyses above with statistical inference procedures that borrow from inductive data analysis and combinatorial approaches (for this part even more than for the others, we refer to Le Roux and Rouanet, 2004 & 2010).

The typicality problem consists in asking whether a group of individuals can be assimilated to the reference population or whether it is atypical. A **typicality test** calculates a *combinatorial p-value*, which defines the "degree of typicality" of the mean point of the group of individuals. A low p-value is considered statistically significant in the combinatorial sense and reflects a difference that is probably not due to chance.

```
dimtypicality(mca, Taste[, c("Gender","Age","Educ")], dim = c(1,2))
```

```
$dim.1
             weight test.stat p.value
Age.15-24       297  7.890052 0.00000
Educ.None       490  6.487410 0.00000
Educ.Low        678  4.487336 0.00001
Age.25-49       849  0.669142 0.50340
```

```
Gender.Women   1042  0.543265 0.58695
Gender.Men      958 -0.543265 0.58695
Educ.Medium     357 -2.272883 0.02303
Age.50+         854 -6.340738 0.00000
Educ.High       475 -9.502876 0.00000

$dim.2
             weight  test.stat p.value
Age.15-24       297  12.875430 0.00000
Gender.Men      958  10.972527 0.00000
Age.25-49       849   5.520379 0.00000
Educ.High       475   5.507427 0.00000
Educ.Low        678   1.503312 0.13276
Educ.Medium     357   0.409932 0.68186
Educ.None       490  -7.468929 0.00000
Gender.Women   1042 -10.972527 0.00000
Age.50+         854 -14.772247 0.00000
```

At a 5 percent threshold, the mean points for women, men, and middle-aged people are not significantly different from the mean point for the entire population on axis 1 (i.e., 0). On axis 2, it is the mean points of the low and middle education levels that do not differ significantly from the origin.

One can study the results of typicality tests for a supplementary variable in particular using the function `supvar()`.

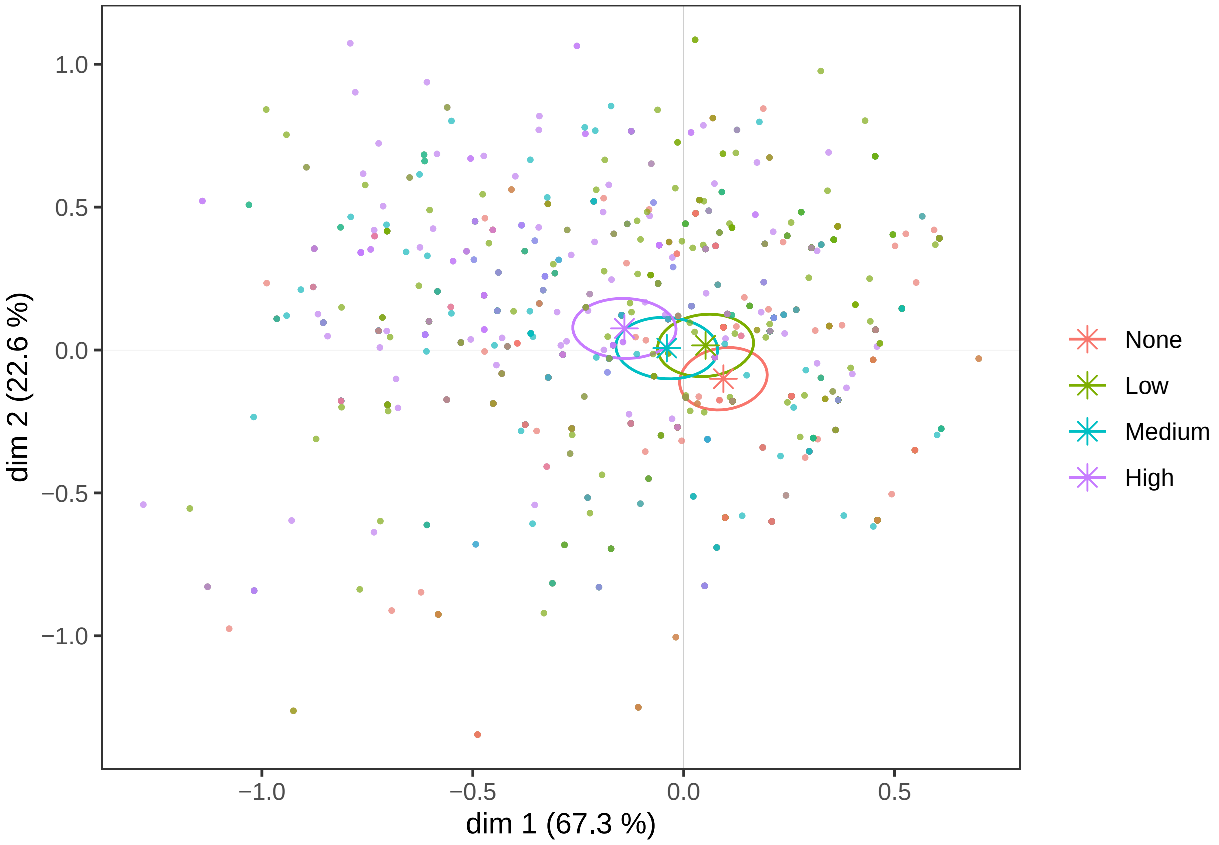
```
vseduc <- supvar(mca, Taste$Educ)
vseduc$pval[, c(1,2)]
```

```
          dim.1    dim.2
None   0.000000 0.000000
Low    0.000007 0.132758
Medium 0.023033 0.681856
High   0.000000 0.000000
```

We see that, on axis 1, all the education level categories are significantly different from 0 at a 5% threshold, they are atypical of the cloud of all individuals, but this is not the case for the “Low” and “Medium” categories on axis 2.

Confidence ellipses follow the same logic as typicality tests. With a conventional significance level of 5%, the confidence ellipse is a 95% confidence zone representing, for a category, the set of possible mean points that are not significantly different from the observed mean point.

```
# p <- ggcloud_indiv(mca, col='lightgrey')
ggadd_ellipses(icloud, mca, Taste$Educ, level = 0.05, label = FALSE)
```



We find here graphically the results obtained with the typicality tests.

A **homogeneity test** is a combinatorial procedure that aims at comparing several groups of individuals. The question is to know if, on a given axis, the positions of two groups of individuals are significantly different (i.e. if the p-values are all very close to 0). The groups correspond to the different categories of a variable.

```
ht <- homog.test(mca, Taste$Educ)
round(ht$dim.1$p.values, 3)
```

```
       None Low Medium High
None      1   0      0    0
Low       0   1      0    0
Medium    0   0      1    0
High      0   0      0    1
```

On axis 1, the mean points of the education level categories are all significantly different from each other.

```
round(ht$dim.2$p.values, 3)
```

```
       None   Low Medium High
None      1 0.000  0.000    0
Low       0 1.000  0.223    0
Medium    0 0.223  1.000    0
High      0 0.000  0.000    1
```

This is not the case on axis 2, where the “Low” and “Medium” categories are not significantly different (p-value = 0.22).

Internal validation

The lack of robustness of MCAs, or the absence of means to measure this robustness, is sometimes presented as one of the weaknesses of geometric data analysis. These criticisms are hardly relevant, especially since there are techniques for the "internal validation" of MCA results. Ludovic Lebart (2006; 2007) has proposed the use of *bootstrap*, i.e. resampling by random draw with replacement. This approach has the advantage of being non-parametric, i.e. it does not rely on any of the probabilistic assumptions of frequentist inference.

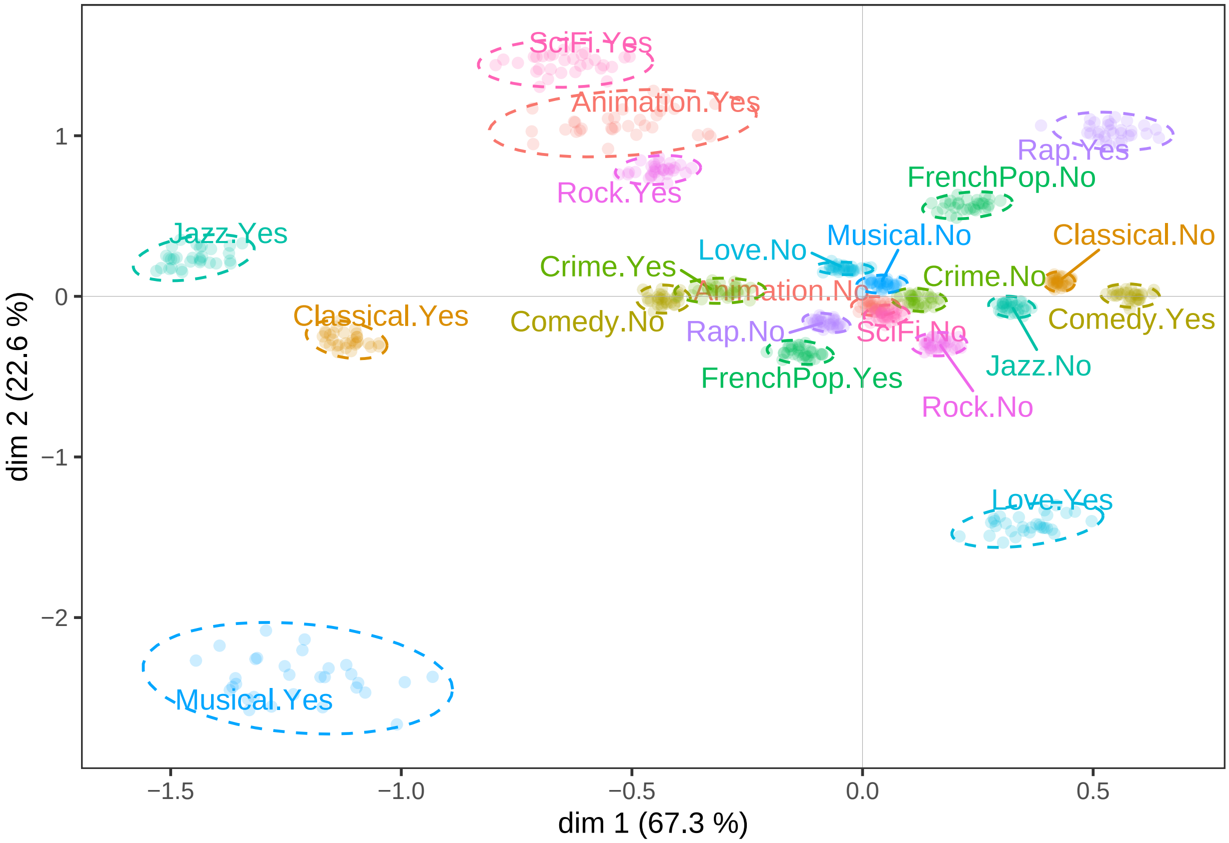
Lebart proposes several methods. The **total bootstrap** uses new MCAs computed from bootstrap replications of the initial data.

- In the *type 1* total bootstrap, the sign of the coordinates is corrected if necessary (the direction of the axes of an ACM being arbitrary).
- In the *type 2*, the order of the axes and the sign of the coordinates are corrected if necessary.
- In *type 3*, a procustean rotation is used to find the best overlap between the initial and replicated axes.

The **partial bootstrap**, on the other hand, does not compute a new MCA: it projects bootstrap replications of the initial data as supplementary elements of the MCA. It gives a more optimistic (or less demanding) view of the stability of the results than the total bootstrap.

Below, the results of a partial bootstrap (with 30 replications, a number generally considered sufficient by Lebart) are represented with confidence ellipses. We can see that if the points of the bootstrap replications of some categories are relatively scattered, this is only the case for categories far from the origin of the plane and does not change the interpretation of the MCA results: they are quite robust.

```
ggbootvalid_variables(mca, type = "partial", K = 30) + theme(legend.position = "none")
```



It is possible to conduct the same type of analysis for supplementary variables with the function `ggbootvalid_supvars()`, and to obtain the results of bootstrap replication calculations (without graphing) with the functions `bootvalid_variables()` and `bootvalid_supvars()`.

MCA and clustering

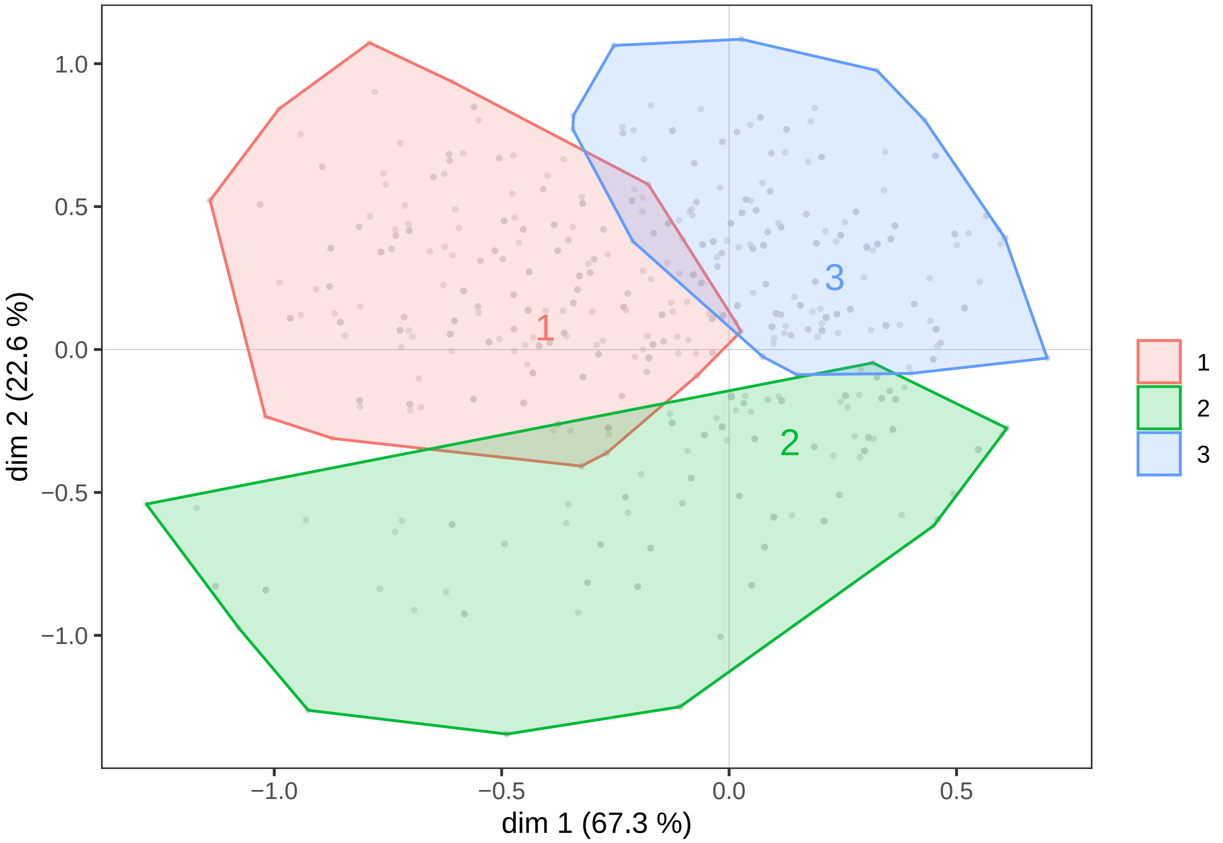
MCA and automatic clustering techniques are often used together. For example, after having represented the data by factorial planes, one may wish to obtain a summary in the form of a typology, i.e. to group together individuals who are similar. To do this, we can use the original data or the coordinates of the individuals in the MCA cloud.

We perform here a **Ascending Hierarchical Classification** (AHC) from the coordinates of the individuals in the factorial plane (1,2), then a partition in 3 clusters.

```
d <- dist(mca$ind$coord[, c(1,2)])
ahc <- hclust(d, "ward.D2")
cluster <- factor(cutree(ahc, 3))
```

The partition thus created can then be used for further analysis, for example by crossing it with supplementary variables, or more simply by representing the subclouds of individuals corresponding to the clusters of the typology, using **convex hulls**. A convex hull is the smallest convex polygon among those containing a set of points. In the context of MCA, convex hulls are an interesting graphical representation, especially if the subclouds represented do not overlap too much, as is the case here.

```
ggadd_chulls(icloud, mca, cluster)
```



Some remarks:

- The function `quadrant()` also creates a typology of individuals, in a simpler way: each individual is classified according to the part of the plane (the "quadrant") where it is located (northwest, northeast, southeast, southwest).

- There are many measures of distance / similarity. In the example above, we used the Euclidean distance between the coordinates of the individuals. If we perform an automatic clustering *from the original variables* of the MCA, it is consistent to use the chi-2 distance, the one used by correspondence analyses, which is possible with the function `dist.chi2()`.
- The function `ahc.plots()` proposes some graphical representations which give indications for the choice of the number of clusters from the results of an AHC.
- If the AHC is the most widespread automatic clustering technique in geometric data analysis, there are many others (see for example the package `cluster`).

MCA variants

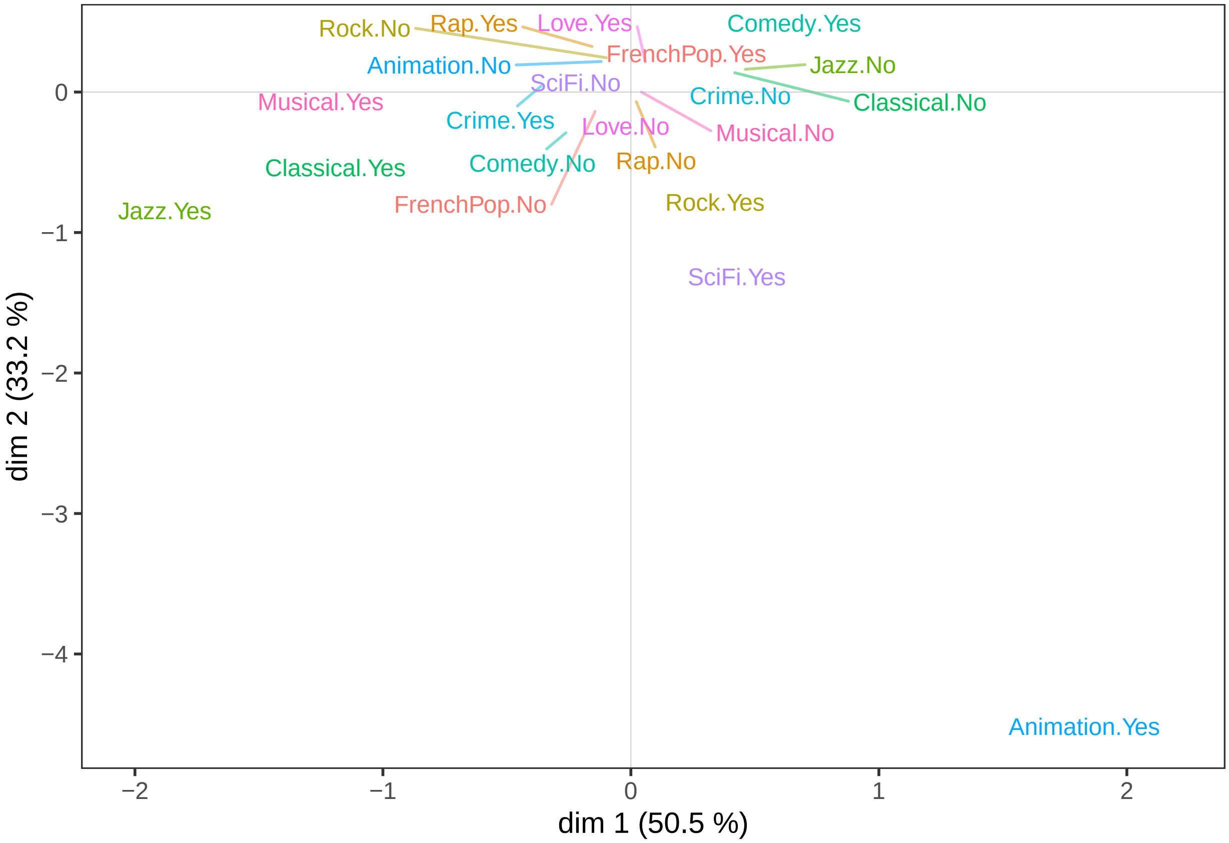
Class Specific Analysis

The **Class Specific Analysis** (CSA) is an extension of MCA which allows to study a subcloud of individuals by taking into account the distribution of variables in the subcloud *and* in the whole cloud. It is thus a question of taking into account the fact that the structure of the subcloud does not exist *in abstracto* but in relation to the cloud in which it is included.

CSA is illustrated here using the subcloud of the most highly educated individuals.

```
csa <- csMCA(Taste[,1:11], Taste$Educ=="High", excl=c(3,6,9,12,15,18,21,24,27,30,33))
```

```
ggcloud_variables(csa, shapes = FALSE, legend = "none")
```



```
tabcontrib(csa, dim = 1)
```

Variable	Category	Weight	Quality of representation	Contribution (left)	Contribution (right)	Total contribution	Cumulated contribution	Contribution of deviation	Proportion to variable
Jazz	Yes	145	0.629	41.85		51.08	51.08	17.25	33.76
	No	326	0.608		9.23				
Classical	Yes	183	0.363	17.79		24.55	75.63	6.9	28.1
	No	290	0.359		6.76				
Animation	Yes	34	0.149		12.43	12.43	88.06	4.64	35.66

We can see that, as in the MCA, listening to jazz and classical music structure axis 1, but this time much more strongly since these two variables alone contribute 75% to the construction of the axis.

```
tabcontrib(csa, dim = 2)
```

Variable	Category	Weight	Quality of representation	Contribution (left)	Contribution (right)	Total contribution	Cumulated contribution	Contribution of deviation	Proportion to variable
Animation	Yes	34	0.651	59.05		59.05	59.05	22.06	35.66
SciFi	Yes	38	0.149	9.51		9.51	68.55	2.53	24.72
Rock	Yes	203	0.118	6.71		6.71	75.26	2.55	27.32
Jazz	Yes	145	0.075	5.43		5.43	80.69	2.16	32.22

It is above all the taste for animation movies that contributes to the construction of axis 2.

In sum, the subcloud of the variables for the most highly educated individuals has points in common with that of the population as a whole, but also some very marked specificities.

The similarity between the MCA on all individuals and the CSA on the most highly educated individuals can be measured by studying the angles between the axes of the two analyses, with the function `angles.csa()`.

```
angles.csa(csa, mca)
```

```
$cosines
         mca.dim1 mca.dim2 mca.dim3 mca.dim4 mca.dim5 mca.dim1&2 mca.dim1&3
csa.dim1    0.772    0.157   -0.051    0.415    0.486      0.787      0.773
csa.dim2    0.553   -0.481   -0.063   -0.511   -0.535      0.733      0.557
csa.dim3   -0.017   -0.680   -0.059    0.010    0.480      0.680      0.061
csa.dim4    0.223    0.182    0.385   -0.482    0.179      0.287      0.445
csa.dim5   -0.181   -0.296   -0.298   -0.350    0.154      0.347      0.348
         mca.dim1&4 mca.dim1&5 mca.dim2&3 mca.dim2&4 mca.dim2&5 mca.dim3&4
csa.dim1      0.876      0.912      0.165      0.444      0.511      0.418
csa.dim2      0.753      0.770      0.485      0.702      0.719      0.515
csa.dim3      0.020      0.480      0.682      0.680      0.832      0.059
csa.dim4      0.531      0.286      0.426      0.515      0.256      0.617
csa.dim5      0.394      0.238      0.420      0.458      0.334      0.459
         mca.dim3&5 mca.dim4&5
csa.dim1      0.489      0.639
csa.dim2      0.539      0.740
csa.dim3      0.483      0.480
csa.dim4      0.425      0.515
csa.dim5      0.335      0.382
```

```
$angles
          mca.dim1 mca.dim2 mca.dim3 mca.dim4 mca.dim5 mca.dim1&2 mca.dim1&3
csa.dim1      39.5     81.0     87.1     65.5     60.9       38.1       39.4
csa.dim2      56.4     61.3     86.4     59.3     57.7       42.9       56.2
csa.dim3      89.0     47.2     86.6     89.4     61.3       47.2       86.5
csa.dim4      77.1     79.5     67.3     61.2     79.7       73.3       63.6
csa.dim5      79.6     72.8     72.7     69.5     81.2       69.7       69.6
          mca.dim1&4 mca.dim1&5 mca.dim2&3 mca.dim2&4 mca.dim2&5 mca.dim3&4
csa.dim1        28.8       24.2       80.5       63.7       59.3       65.3
csa.dim2        41.1       39.7       61.0       45.4       44.0       59.0
csa.dim3        88.9       61.3       47.0       47.2       33.7       86.6
csa.dim4        57.9       73.4       64.8       59.0       75.2       51.9
csa.dim5        66.8       76.3       65.2       62.7       70.5       62.7
          mca.dim3&5 mca.dim4&5
csa.dim1        60.7       50.2
csa.dim2        57.4       42.3
csa.dim3        61.1       61.3
csa.dim4        64.9       59.0
csa.dim5        70.4       67.5
```

We can see that the first dimension of the CSA is clearly correlated to the first dimension of the MCA (and very little to the second dimension), but that the second dimension of the CSA is quite distinct from the first two dimensions of the MCA.

We will not go further here but let us specify that all the visualization and interpretation techniques described previously can be applied to the results of a CSA.

Discriminant analysis

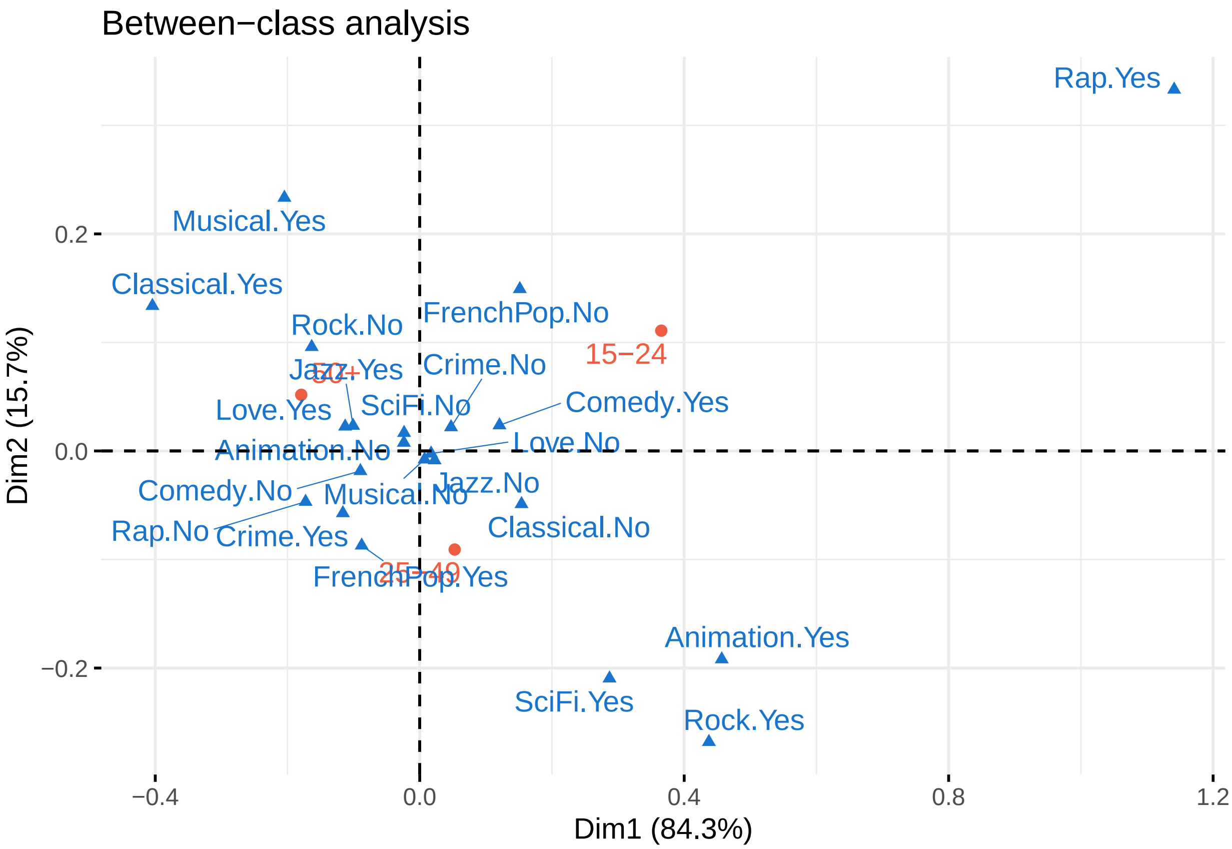
The instrumental variables approach is a form of approximation between geometric data analysis and regression methods. In practice, it consists of constructing a factorial space, from a first group of active variables, so that this space "explains" a second set of variables, called **instrumental variables**, as well as possible. The term "explain" is used here in a sense similar to analysis of variance (anova) or regression, in which an explanatory variable "explains" an explained variable, i.e. accounts for its variance as well as possible.

In the general case, there are several instrumental variables, which can be categorical and/or numerical. When the active variables are all numerical, we speak of Principal Component Analysis on Instrumental Variables (function `PCAiv()`), and when the active variables are all categorical, of Correspondence Analysis on Instrumental Variables (function `MCAiv()`).

If the instrumental variable is unique and categorical, we can say that it partitions the individuals into groups or "classes". It is therefore a question of carrying out an **inter-class analysis**, by constructing a factorial space which accounts for the differences between classes (functions `bcPCA()` for cases where the active variables are continuous and `bcMCA()` for those where they are categorical). It is also sometimes called **barycentric discriminant analysis** or *discriminant correspondence analysis*. If the active variables are continuous, a variant is the **factorial discriminant analysis** (FDA), also called *descriptive discriminant analysis* (function `DA()`), which is equivalent to *linear discriminant analysis* in the Anglo-Saxon literature. When the active variables are categorical, an adaptation of the FDA has been proposed by Gilbert Saporta (1977) under the name **Disqual** (function `DAQ()`).

As an example, we apply a barycentric discriminant analysis to construct the factorial space of the differences in tastes between individuals of different age categories.

```
between <- bcMCA(Taste[,1:11], class = Taste$Age, excl = junk)
factoextra::fviz_ca(between, repel = TRUE, invisible = "row.sup",
                    col.col = "dodgerblue3", col.row = "tomato2", title = "Between-class analysis")
```



```
between$ratio
```

```
[1] 0.04077871
```

The first axis, or "discriminant factor", orders the age categories and in particular contrasts listening to rap and rock music and taste for science fiction, on the side of the young, with listening to classical music and taste for musicals, on the side of the over 50s. But many of the taste categories are close to the center of axis 1 and are therefore not differentiated much by age. Age differences account for only 4% of the total inertia of the cloud.

Conditional analysis

Conditional analyses are another attempt to integrate geometric data analysis and regression. Their principle is to constrain the axes of the MCA to be independent (i.e. orthogonal) of one or more supplementary variables, i.e. to construct a MCA "all things (of these supplementary variables) equal" (Bry et al, 2016). Comparing the results of the original MCA with those of the conditional MCA is a way to study structure effects.

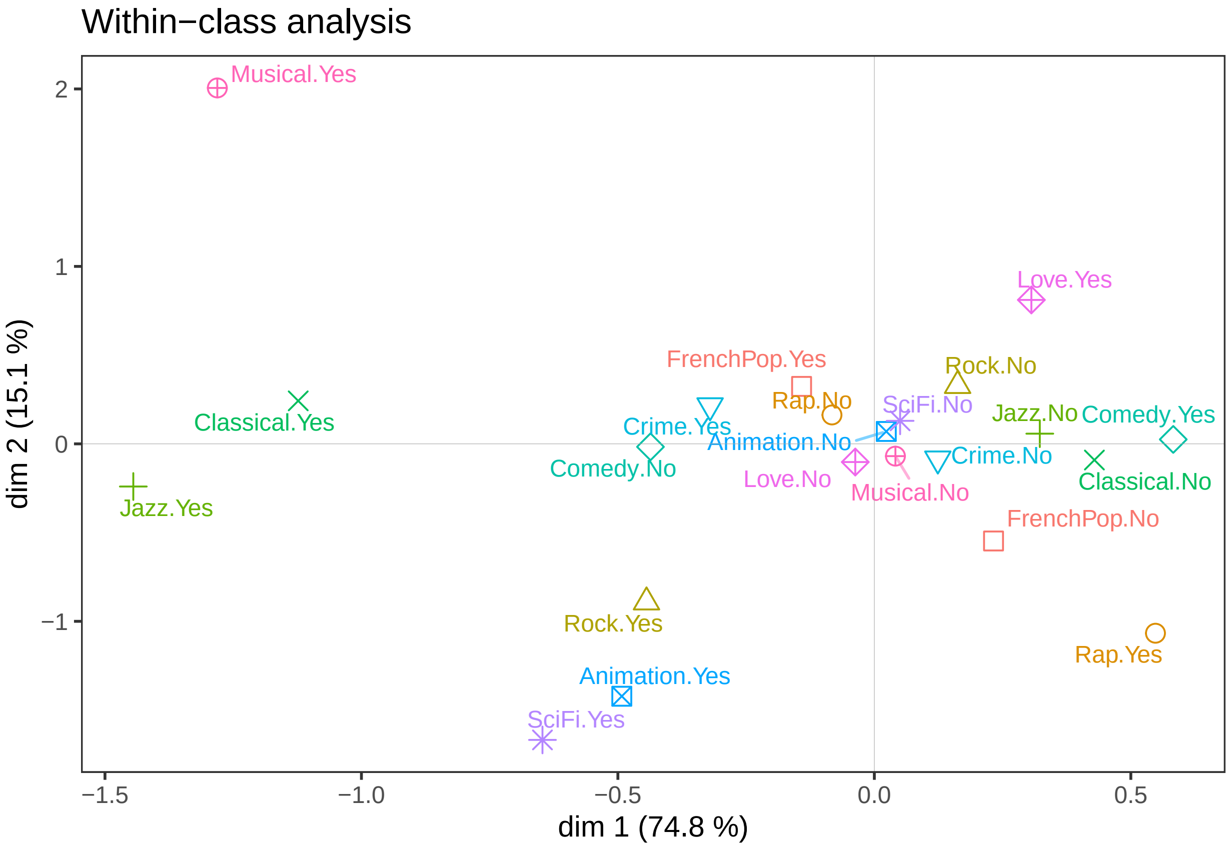
In the general case, there are several variables whose effect must be "eliminated" and these are sometimes called **orthogonal instrumental variables**. We will therefore carry out a *Principal Component Analysis on Orthogonal Instrumental Variables* if the active variables are all numerical (function `PCAoiv()`) and a *Correspondence Analysis on Orthogonal Instrumental Variables* if the active variables are all categorical (function `MCAoiv()`).

As for the discriminant analyses (previous sub-section), if the orthogonal instrumental variable is unique and categorical, it partitions the individuals into "classes". But this time we are in the context of an **intra-class analysis**: the aim is to construct the factorial space that accounts for the structures common to the different classes. When the active variables are categorical, a **conditional MCA** (Escofier, 1990 and function `wcMCA()`) is performed, of which the **standardized MCA** (Bry et al, 2016) is an alternative (function `stMCA()`). The conditional approach also applies to the case of numerical active variables (function `wcPCA()`).

We now construct the space of tastes common to women and men, or, put differently, neutralizing the gendered differences. In the plane(1,2), the results obtained are very similar to those of the MCA. The main difference concerns the (very female) taste for love movies, which is no longer distinguishable on axis 2.

The small size of the differences between the two analyses is confirmed by the fact that the within-class analysis accounts for almost 99% of the total inertia of the cloud.

```
within <- wcMCA(Taste[,1:11], class = Taste$Gender, excl = junk)
ggcloud_variables(within, legend = "none") + ggtitle("Within-class analysis")
```



```
within$ratio
```

```
[1] 0.9896936
```

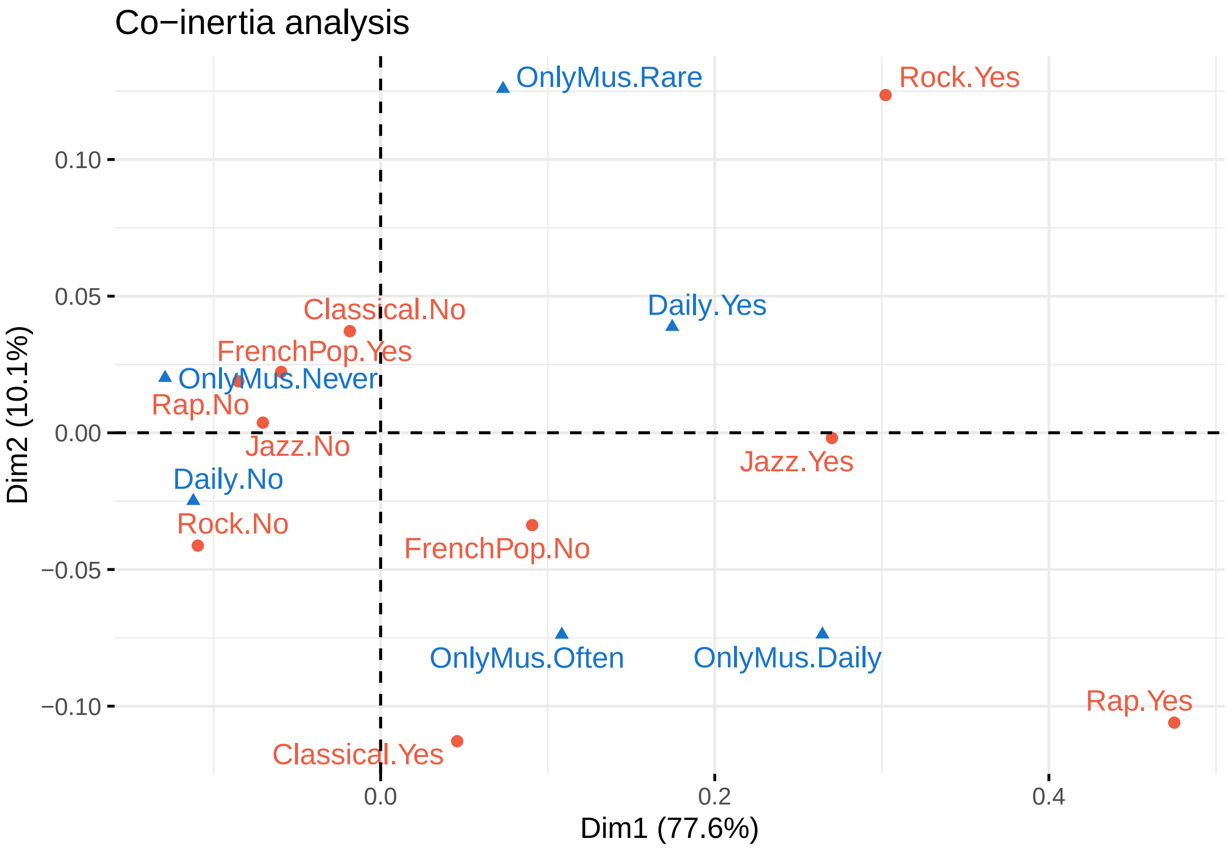
Symmetrical multi-table analysis

Another case arises when the active variables are divided into several homogeneous groups and these groups play a symmetrical role (unlike the approaches with instrumental variables).

With two groups of variables, one can perform a **co-inertia analysis** (Tucker, 1958; Dolédec and Chessel, 1994) in order to study the structures common to both groups (functions `coiMCA()` if the groups are composed of categorical variables, `coiPCA()` if they are composed of numerical variables).

A co-inertia analysis is applied to a new dataset, in which a group of 5 variables describes the tastes for different musical genres, and a second group of 2 variables describes the frequency of listening (in general or "focused", i.e. doing nothing else).

```
data(Music)
Xa <- Music[,1:5]  # music tastes
Xb <- Music[,8:9]  # frequency of listening
coin <- coiMCA(Xa, Xb,
               excl.a = c("FrenchPop.NA","Rap.NA","Rock.NA","Jazz.NA","Classical.NA"))
factoextra::fviz_ca(coin, repel = TRUE, title = "Co-inertia analysis",
                    col.col = "dodgerblue3", col.row = "tomato2")
```

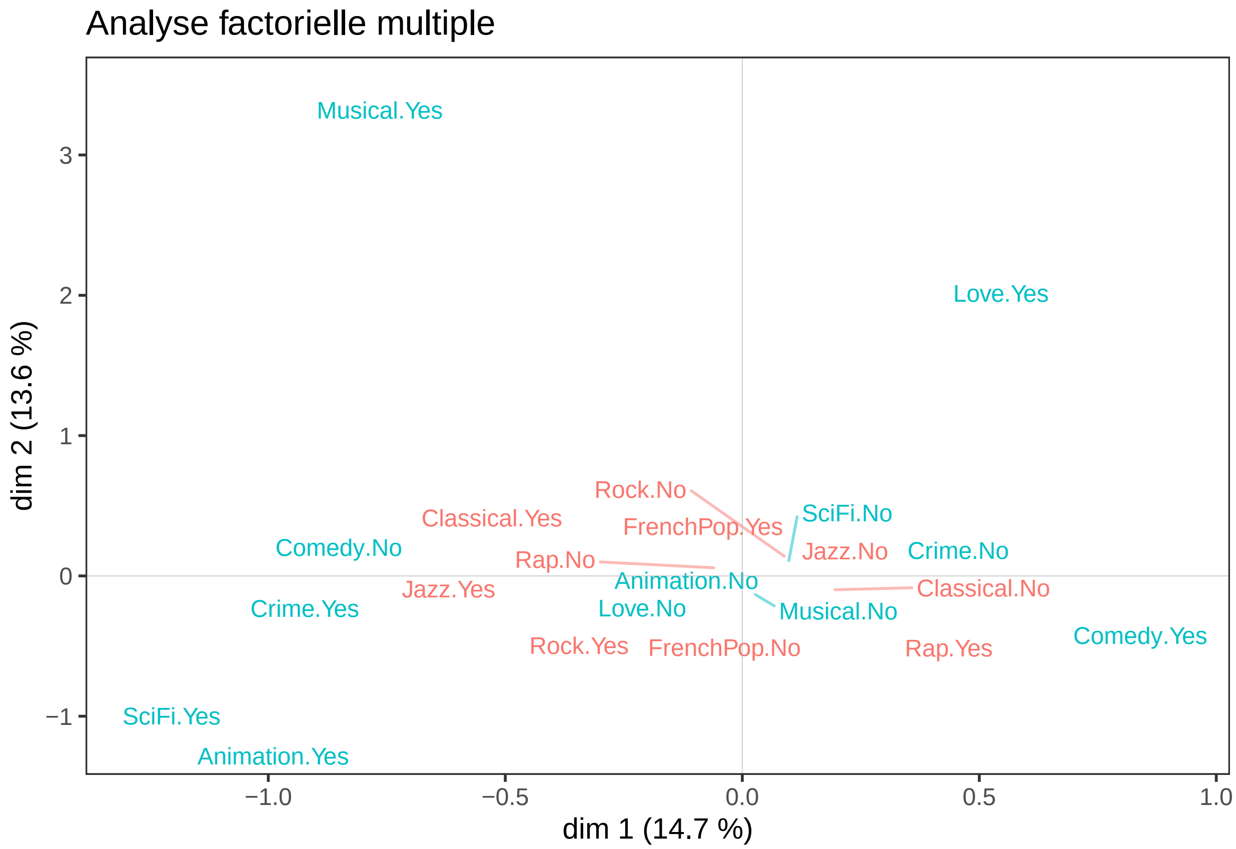


Examination of the first axis shows that rap, jazz and rock are on the side of daily listening and frequent or daily focused listening, while taste for French popular music is on the side of non-daily listening and no focused listening.

Multiple Factor Analysis (MFA, see Escofier and Pagès, 1994) makes it possible to deal with two groups of variables *or more*. It takes into account both the structures specific to each group and the structures common to the groups. The function `multiMCA()` makes it possible to carry out MFA using specific MCA or Class Specific Analysis.

Here, we use the `Taste` data. One group of variables describes the listening to music genres and the other the taste for movie genres, and the MFA uses specific MCAs.

```
mca1 <- speMCA(Taste[,1:5], excl = c(3,6,9,12,15))
mca2 <- speMCA(Taste[,6:11], excl = c(3,6,9,12,15,18))
mfa <- multiMCA(list(mca1,mca2))
ggcloud_variables(mfa, shapes=FALSE, legend="none") + ggtitle("Analyse factorielle multiple")
```



We observe that it is mainly the film variables that structure the plane (1,2). An MFA is of little interest here, because the two spaces are very little related: the RV coefficient is 1.4% (the **RV coefficient** - function `rvcoef()` - is a kind of correlation coefficient between groups of variables).

```
rvcoef(mca1$ind$coord, mca2$ind$coord)
```

```
[1] 0.01412768
```

NB: There is another approach to deal with the case of several groups of variables, whether these groups play a symmetrical role or not, it is the **PLS approach** (Partial Least Square, see Tenenhaus, 1998). Most of these techniques are available in the package `plsdepot`, developed by Gaston Sanchez.

Nonsymmetric correspondence analysis

When dealing with a contingency table with a dependence structure, i.e. when the role of the two variables is not symmetrical but, on the contrary, one can be considered as predicting the other, nonsymmetric correspondence analysis (NSCA) can be used to represent the predictive structure in the table and to assess the predictive power of the predictor variable.

Technically, NSCA is very similar to the standard CA, the main difference being that the columns of the contingency table are not weighted by their rarity (i.e. the inverse of the marginal frequencies).

Here we study data on the distribution of various domestic tasks in couples. We consider that the type of domestic task is predictive of *who* performs that task.

```
library(ade4)
data(housetasks)
housetasks
```

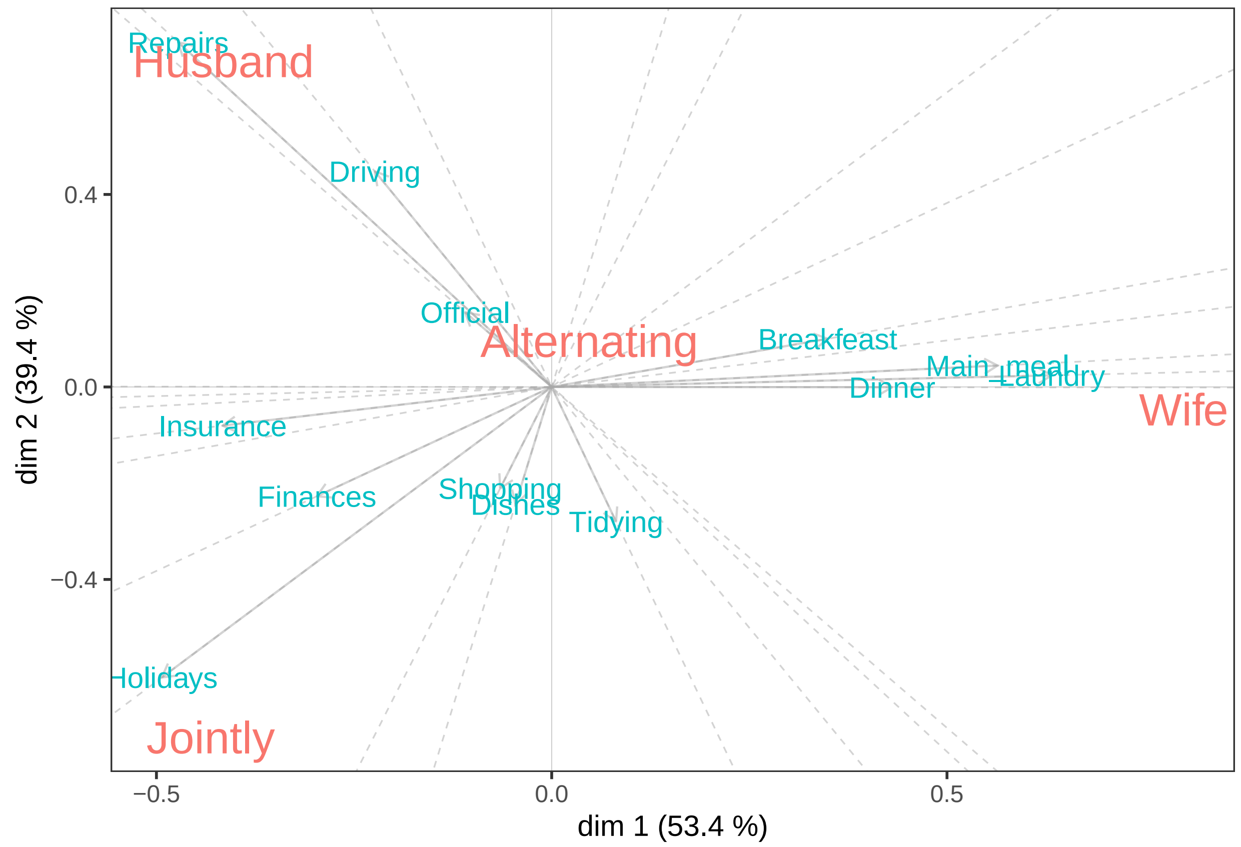
```
           Wife Alternating Husband Jointly
```

```
Laundry      156           14       2        4
Main_meal    124           20       5        4
Dinner        77           11       7       13
Breakfeast    82           36      15        7
Tidying       53           11       1       57
Dishes        32           24       4       53
Shopping      33           23       9       55
Official      12           46      23       15
Driving       10           51      75        3
Finances      13           13      21       66
Insurance      8            1      53       77
Repairs        0            3     160        2
Holidays       0            1       6      153
```

The biplots of an NSCA reflect the dependency structure of the contingency table and thus should not be interpreted as the planes of a standard CA. A first principle is that the graph displays the centred row profiles. A second principle is that the relationships between rows and columns are contained in their inner products : the rows are depicted as vectors, also called biplot axes, and the columns are projected on these vectors. If some columns have projections on the row vector far away from the origin, then the row has a comparatively large increase in predictability, and its profile deviates considerably from the marginal one, especially for that column.

For more detailed interpretational guidelines, see Kroonenberg and Lombardo (1999, pp.377-378).

```
nsca <- nsCA(housetasks)
nsca.biplot(nsca)
```



```
nsca$GK.tau
```

```
[1] 0.4067139
```

We find that knowing that household tasks involve laundry or meal preparation "increases predictive performance" of female tasks. Repairs are associated with male tasks and vacation planning with joint tasks.

The **Goodman and Kruskal tau** is an *asymmetric* measure of global association. Its high value (0.41) here indicates that the type of household task does predict who performs the task.

Some practical points

Links with other packages

The MCA and PCA variants offered in GDAtools are, as far as possible, intended to create objects similar to those created by the `PCA()`, `MCA()` and `CA()` functions of the [`FactoMineR`] package (https://cran.r-project.org/package=FactoMineR). This allows these objects to be (mostly) compatible with a number of functions for interpreting and visualizing the results:

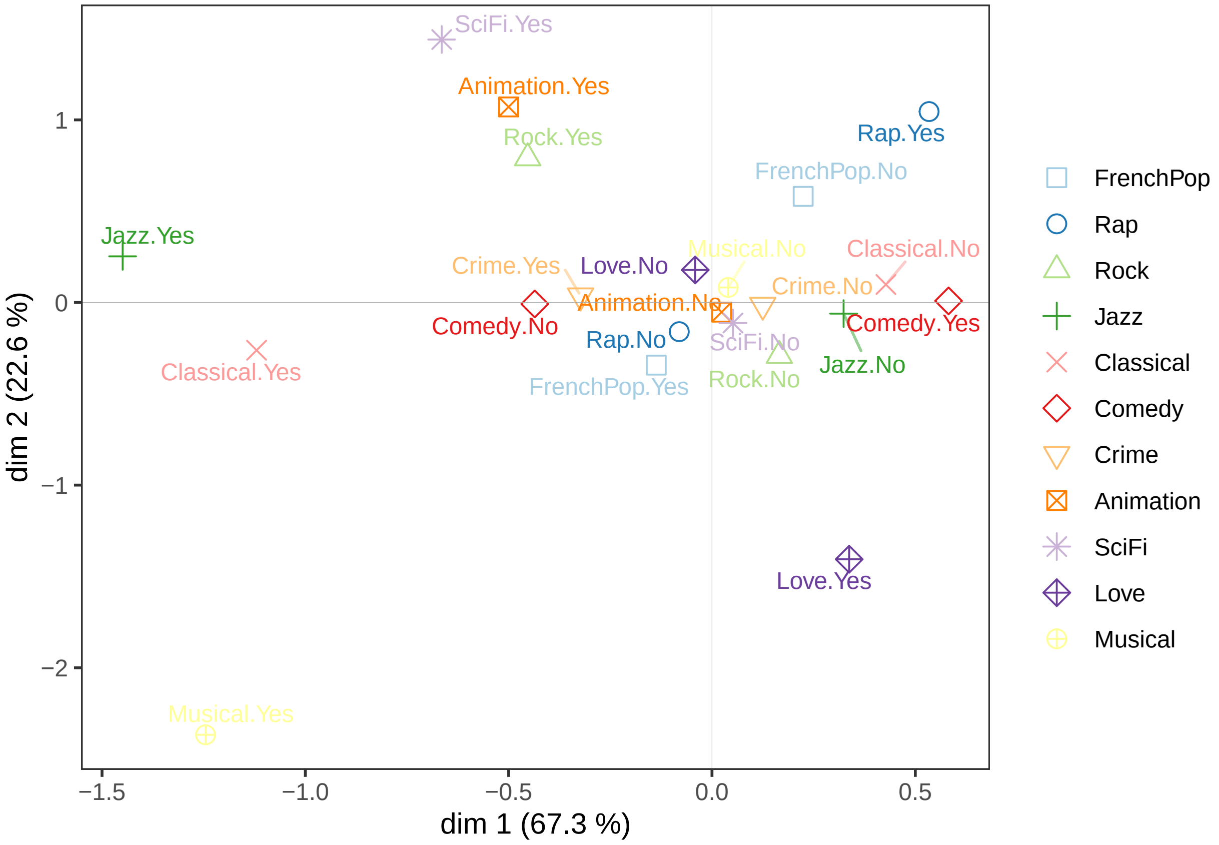
- the graphical functions of the `FactoMineR` package,
- the package `Factoshiny` for the interactive construction of graphs,
- the amazing package `explor`, developed by Julien Barnier, for interactive exploration of results,
- the package `factoextra` to extract and visualize the results.

Color customization

Most of the graphical functions in `GDAtools` have been designed with the "grammar" (and default color palettes) of `ggplot2` in mind, and in such a way as to be able to customize colors in a very flexible way. One can indeed use `scale_color_*` functions, like `scale_color_grey()` for gray scales, `scale_color_brewer()` for `ColorBrewer` palettes, `scale_color_manual()` for using custom palettes, or `paletteer::scale_color_paletteer_d()` from the package `paletteer` for accessing a large number of color palettes existing in R (a list of which is available here). However, one must be careful to choose a palette with at least as many colors as there are variables to represent (otherwise some variables will remain invisible).

For example, to display the cloud of variables with the `ColorBrewer` palette "Paired1", we will proceed as follows.

```
ggcloud_variables(mca) + scale_color_brewer(palette = "Paired")
```


SciFi.Yes
Animation.Yes
Rock.Yes
Rap.Yes
FrenchPop.No
Jazz.Yes
Musical.No
Classical.No
Crime.Yes
Love.No
Crime.No
Animation.No
Comedy.Yes
Comedy.No
Rap.No
SciFi.No
Jazz.No
Classical.Yes
FrenchPop.Yes
Rock.No
Love.Yes
Musical.Yes
1
0
−1
−2
−1.5
−1.0
−0.5
0.0
0.5
dim 1 (67.3 %)
dim 2 (22.6 %)
□ FrenchPop
○ Rap
△ Rock
+ Jazz
× Classical
◇ Comedy
▽ Crime
☒ Animation
✳ SciFi
◈ Love
⊕ Musical